Supplementary Material for Commitment vs. Flexibility

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This supplementary document collects two results. First, we cover some findings regarding the possibilities for money burning with three types. Second, we present a result on how simple minimum savings allocations can be improved upon if Assumption A in the paper fails.

1 Money Burning with Three Types

In this section, we study the optimality of money burning when there are only three possible shocks. Our main result concerns the case when the probability of the middle shock vanishes. We also report some numerical findings for higher values of p_m.

Let $\Theta = \{\theta_l, \theta_m, \theta_h\}$ with $\theta_l < \theta_m < \theta_h$. The problem is given by

$$\max \sum_{s\in\{l,m,h\}} [\theta_s U(c_s) + W(k_s)]p_s$$

subject to

$$\begin{aligned} c_s + k_s &\leq y \quad \text{for } s \in \{l, m, h\} \\ \theta_l U(c_l) + \beta W(k_l) &\geq \theta_l U(c_m) + \beta W(k_m) \\ \theta_m U(c_m) + \beta W(k_m) &\geq \theta_m U(c_h) + \beta W(k_h) \\ c_l \geq c_m &\geq c_h \end{aligned}$$

Let (c_s^*, k_s^*) represent the first best allocation for given s, which is independent of the probabilities (p_h, p_m, p_l).

The full parameters of the problem are $(\beta,\ \theta_l,\ \theta_m,\ \theta_h,\ p_h,\ p_m,\ p_l)$. To state our result we consider all families of problems that are indexed by p_m as follows. Let $p_h(p_m)$ and $p_l(p_m)$ be continuous functions such that $p_h(p_m) + p_l(p_m) + p_m = 1$ with $\lim_{p_m \to 0} p_h(p_m) \in (0, 1)$. The following conditions guarantee the optimality of money burning for small enough p_m.

Proposition 1. *There exists a $\bar{p}_m > 0$ such that the optimal allocation of the problem with parameters $(\beta, \theta_l, \theta_m, \theta_h, p_h(p_m), p_m, p_l(p_m))$ has $c_m + k_m < y$ for $0 < p_m \leq \bar{p}_m$ if*

(i) $\beta < \theta_l / \theta_m$

(ii) $\beta > \beta^*$, *i.e. the first-best allocation is such that*

$$\theta_l U(c_l^*) + \beta W(k_l^*) > \theta_l U(c_h^*) + \beta W(k_h^*)$$

(iii) the $(\hat{c}, \hat{k})$ *defined by*

$$\theta_l U(c_l^*) + \beta W(k_l^*) = \theta_l U(\hat{c}) + \beta W(\hat{k}) \tag{1}$$

$$\theta_m U(\hat{c}) + \beta W(\hat{k}) = \theta_m U(c_h^*) + \beta W(k_h^*) \tag{2}$$

is such that $\hat{c} + \hat{k} < y$.

Conversely, if any of the inequalities in conditions (i)–(iii) are reversed, then money burning cannot be optimal for small enough p_m (i.e. there does not exist such a $\bar{p}_m$).

Proof. An allocation (c_s, k_s) with $c_l > c_m > c_h$ is optimal if and only if it is feasible and there exists non-negative multipliers such that the first-order conditions hold:

$$(p_l + \mu_l)\theta_l U'(c_l) = \lambda_l \tag{3}$$

$$(p_l + \beta\mu_l) W'(k_l) = \lambda_l \tag{4}$$

$$\left(p_m - \frac{\theta_l}{\theta_m}\mu_l + \mu_m\right)\theta_m U'(c_m) = \lambda_m \tag{5}$$

$$\left(p_m - \beta\mu_l + \beta\mu_m\right) W'(k_m) = \lambda_m \tag{6}$$

$$\left(p_h - \frac{\theta_m}{\theta_h}\mu_m\right)\theta_h U'(c_h) = \lambda_h \tag{7}$$

$$\left(p_h - \beta\mu_m\right) W'(k_h) = \lambda_h \tag{8}$$

Where λ_s are the Lagrange multipliers on the resource constraints, and μ_s are the multipliers on the incentive constraints. In addition, we require the usual complementary slackness conditions, i.e. that the multipliers are zero if the associated inequalities are strict.

For the sufficiency part, we proceed by explicitly constructing an allocation as a function of the probability p_m. We then show that, for low enough p_m, the constructed allocation is optimal and has money burning. The allocation we construct satisfies the first-order conditions (3)–(8), has the resource constraints binding for the low and high types, and imposes the complementary slackness condition that $\lambda_m(p_m) = 0$.

Using (5) and (6) we can now solve for the multipliers

$$\mu_l(p_m) = \frac{1-\beta}{\beta}\frac{1}{1-\theta_l/\theta_m}p_m \tag{9}$$

$$\mu_m(p_m) = \frac{1}{\beta}\left(\frac{\theta_l/\theta_m - \beta}{1-\theta_l/\theta_m}\right)p_m \tag{10}$$

which are positive since $\theta_l/\theta_m - \beta \geq 0$ and $\beta < 1$. Either equation (3) or equation (4) imply that $\lambda_l(p_m)$ is strictly positive. Since $\mu(p_m)$ goes to zero as p_m goes to zero, both equation (7) and equation (8) require that $\lambda_h(p_m)$ be strictly positive for small enough p_m.

Hence, for small enough p_m we can rearrange the first-order conditions (3), (4), (7) and (8) as

$$\frac{\theta_l U'(c_l(p_m))}{W'(k_l(p_m))} = \frac{p_l(p_m) + \beta\mu_l(p_m)}{p_l(p_m) + \mu_l(p_m)}, \tag{11}$$

$$\frac{\theta_h U'(c_h(p_m))}{W'(k_h(p_m))} = \frac{p_h(p_m) - \beta\mu_m(p_m)}{p_h(p_m) - \frac{\theta_m}{\theta_h}\mu_m(p_m)}, \tag{12}$$

which together with the binding resource constraints $c_l(p_m)+k_l(p_m) = y$ and $c_h(p_m)+k_h(p_m) = y$ can be solved uniquely for $c_l(p_m)$, $k_l(p_m)$, $c_h(p_m)$ and $k_h(p_m)$, and are continuous functions of p_m.

Since $\mu_l(p_m), \mu_m(p_m) > 0$, we solve for $c_m(p_m), k_m(p_m)$ from the binding incentive constraints

$$\theta_l U(c_l(p_m)) + \beta W(k_l(p_m)) = \theta_l U(c_m(p_m)) + \beta W(k_m(p_m)), \tag{13}$$

$$\theta_m U(c_m(p_m)) + \beta W(k_m(p_m)) = \theta_m U(c_h(p_m)) + \beta W(k_h(p_m)). \tag{14}$$

Note that $c_m(p_m)$ and $k_m(p_m)$ are continuous in p_m.

Equations (9)–(10) imply that as $p_m \to 0$ we have that $\mu_l(p_m) \to 0$ and $\mu_m(p_m) \to 0$.

Equations (11)–(12) imply that as $p_m \to 0$,

$$(c_l(p_m), k_l(p_m)) \to (c_l^*, k_l^*) \tag{15}$$

$$(c_h(p_m), k_h(p_m)) \to (c_h^*, k_h^*) \tag{16}$$

since $\lim_{p_m \to 0} p_h(p_m) > 0$ and $\lim_{p_m \to 0} p_l(p_m) > 0$. Continuity of $(c_m(p_m), k_m(p_m))$ implies that, as $p_m \to 0$,

$$(c_m(p_m), k_m(p_m)) \to (\hat{c}, \hat{k}),$$

so that for sufficiently low p_m there is money burning. Finally, part (ii) and (iii) imply that $c_l^* < \hat{c} < c_h^*$, so that indeed, for small enough p_m, the monotonicity constraints are slack: $c_l(p_m) < c_m(p_m) < c_h(p_m)$.

Summarizing, for sufficiently low p_m the constructed allocation is feasible, the monotonicity condition is slack, and all the first-order conditions are met; hence, it is optimal.

The converse statement follows from the fact that the allocation we constructed above is the only one consistent with optimality and the hypothesis of money burning for sufficiently small p_m. Hence, if condition (i) is reversed then $\mu_m(p_m)$ is strictly negative; if condition (ii) is reversed then, since the allocation must satisfy (15)-(16), it cannot be incentive compatible for low p_m; if the inequality in condition (iii) is reversed then, since the allocation must satisfy (1), the resource constraint for the middle type cannot be met for small p_m. *Q.E.D.*

We have also verified numerically that money burning is possible for high enough p_m for cases when condition (ii) in the previous proposition is violated. A concrete example yielding money burning has the following parameter values:

$$\beta = .7,\ \theta_h = 1.6,\ \theta_m = 1,\ \theta_l = 0.8,\ p_l/p_h = .7,\ y = 1$$

with $U(c) = -c^{-1}$. This example is illustrated in Figure 1, which was produced by the Matlab code named `burn.m` provided in Appendix B.[1]

[1] This code produces two graphs showing the regions of p_m where money burning can be possible. The first graph plots the allocations of consumption given to each of the three types. The second one shows the expenditure allocated to the middle type $c_m + k_m$, as in the figure reproduced here.

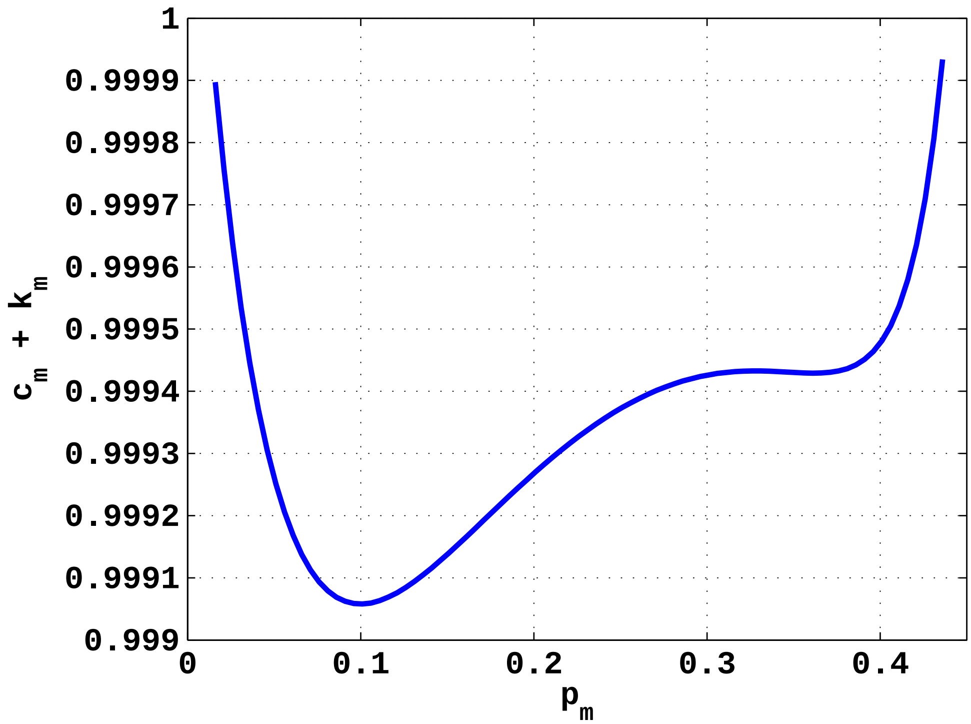

Figure 1—Total expenditure for middle type, $c_m + k_m$, as a function of the probability p_m.

2 Drilling Result

In this section we show that, for the model with a continuous distribution of types, if Assumption A is violated we can improve upon the proposed minimum savings allocation described by Proposition 3 in the paper. The improvement involves removing ("drilling") intervals previously offered.

Suppose we are offering the unconstrained optimum for some closed interval $[\theta_a, \theta_b]$ of agents and we consider removing the open interval (θ_a, θ_b). Agents that previously found their tangency within the interval will move to one of the two extremes, θ_a or θ_b. The critical issue in evaluating the change in welfare is counting how many agents moving to θ_a versus θ_b. For a small enough interval, welfare rises from those moving to θ_a and falls from those moving to θ_b.

Since the relative measure of agents moving to the right versus the left depends on the slope of the density function this explains its role in assumption A. For example, if $f' > 0$ then upon removing (θ_a, θ_b) more agents would move to the right than the left. As a consequence, such a change is undesirable. The proof of the next result formalizes these ideas.

Let $\theta_{ind} \in [\theta_a, \theta_b]$ be the agent type that obtains the same utility from reporting θ_a

or θ_b. We find it more convenient to state the next result in terms of the consumption allocation $c(\theta)$ and $k(\theta)$.

Proposition 2. *Suppose a feasible allocation has $c(\theta) = c^{flex}(\theta)$ and $k(\theta) = k^{flex}(\theta)$ for $\theta \in [\theta_a, \theta_b]$, where $\theta_b \leq \theta_p$. Then if $G(\theta)$ is decreasing on $[\theta_a, \theta_b]$ the alternative allocation*

$$\tilde{c}(\theta), \tilde{k}(\theta) = \begin{cases} c(\theta), k(\theta) & \text{for } \theta \notin [\theta_a, \theta_b] \\ c(\theta_a), k(\theta_a) & \text{for } \theta \in (\theta_a, \theta_{ind}) \\ c(\theta_b), k(\theta_b) & \text{for } \theta \in [\theta_{ind}, \theta_b) \end{cases}$$

increases the objective function and remains feasible.

Proof. Suppose that we are offering a segment of the budget line between the tangency point for θ_L and that of θ_H, with associated allocation c_L and c_H. Define the θ^* that is indifferent from the allocation c_L and c_H then $\theta^* \in (\theta_L, \theta_H)$ for $\theta_H > \theta_L$. Upon removing the interval $\theta \in (\theta^*, \theta_H)$ types move to c_H and $\theta \in (\theta_L, \theta^*)$ types move to c_L allocation.

Let $\Delta(\theta_H, \theta_L)$ be the change in utility for the principal of such a move (normalizing income to $y = 1$ for simplicity)

$$\begin{aligned} \Delta(\theta_H, \theta_L) \equiv & \int_{\theta^*(\theta_H,\theta_L)}^{\theta_H} \{\theta U(c^*(\theta_H)) + W(y - c^*(\theta_H))\} f(\theta) d\theta \\ & + \int_{\theta_L}^{\theta^*(\theta_H,\theta_L)} \{\theta U(c^*(\theta_L)) + W(y - c^*(\theta_L))\} f(\theta) d\theta \\ & - \int_{\theta_L}^{\theta_H} \{\theta U(c^*(\theta)) + W(y - c^*(\theta))\} f(\theta) d\theta \end{aligned}$$

where the function $c^*(\theta)$ is defined implicitly by

$$\theta U'[c^*(\theta)] = \beta W'(y - c^*(\theta)) \tag{17}$$

and $\theta^*(\theta_H, \theta_L)$ is then defined by

$$\theta^*(\theta_H, \theta_L) U(c^*(\theta_H)) + \beta W(y - c^*(\theta_H)) = \theta^*(\theta_H, \theta_L) U(c^*(\theta_L)) + \beta W(y - c^*(\theta_L)) \tag{18}$$

Notice that $\Delta(\theta_L, \theta_L) = 0$.

The partial of $\Delta(\theta_H, \theta_L)$ with respect to θ_H can be expressed as:

$$\frac{\partial \Delta}{\partial \theta_H}(\theta_H, \theta_L) = S(\theta_H; \theta^*) \frac{U'(c^*(\theta_H))}{\beta} \frac{\partial c^*(\theta_H)}{\partial \theta_H}$$

where $S(\theta;\theta^*)$ is defined by,

$$S(\theta,\theta^*) \equiv (y-\beta)(\theta-\theta^*)\theta^* f(\theta^*) - \int_{\theta^*}^{\theta} (\theta - \beta\tilde{\theta}) f(\tilde{\theta}) d\tilde{\theta}$$

Since $U'(c^*(\theta_H)) > 0$ and $\frac{\partial c^*(\theta_H)}{\partial \theta_H} > 0$, then $sign(\Delta_1) = sign(S(\theta_H, \theta^*))$. This result is shown in Appendix A.

We only need to sign $S(\theta_H,\theta^*)$. Clearly, $S(\theta^*,\theta^*) = 0$. Taking derivatives we also get that

$$\frac{\partial S(\theta,\theta^*)}{\partial \theta} = [1-\beta]\theta^* f(\theta^*) - (1-\beta)\theta f(\theta) - \int_{\theta^*}^{\theta} f(\tilde{\theta}) d\tilde{\theta}$$

Notice that

$$\left.\frac{\partial S(\theta,\theta^*)}{\partial \theta}\right|_{\theta^*} = 0$$

$$\frac{\partial^2 S(\theta,\theta^*)}{(\partial\theta)^2} = -(2-\beta) f(\theta) - (1-\beta)\theta f'(\theta)$$

Note that $\partial^2 S(\theta,\theta^*)/(\partial\theta)^2$ does not depend on θ^*, just on θ. It follows that

$$sign\left(\frac{\partial^2 S(\theta,\theta^*)}{(\partial\theta)^2}\right) \leq 0$$

if and only if

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta} \qquad (19)$$

That is, if A holds. Integrating $\partial^2 S(\theta,\theta^*)/(\partial\theta)^2$ twice:

$$S(\theta_H,\theta^*) = \int_{\theta^*}^{\theta_H} \int_{\theta^*}^{\theta} \frac{\partial^2 S(\tilde{\theta},\theta^*)}{(\partial\tilde{\theta})^2} d\tilde{\theta} d\theta$$

Thus $S(\theta_H,\theta^*) \leq 0$ if A holds.

This implies then that $\Delta_1(\theta,\theta_L) \leq 0$ for all $\theta \geq \theta_L$ if assumption A holds; and

$$\Delta(\theta_H,\theta_L) = \int_{\theta_L}^{\theta_H} \Delta_1(\theta;\theta_L) d\theta$$

so that

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta} \Rightarrow \Delta(\theta_H,\theta_L) \leq 0 \;\; ; \text{for all } \theta_H \;\text{ and }\; \theta_L$$

and clearly $\theta_L \in \arg\max_{\theta_H \geq \theta_L} \Delta(\theta_H,\theta_L)$. In other words if assumption A holds then punching holes into any offered interval is not an improvement.

The converse is also true: if A does not hold for some open interval $\theta \in (\theta_1, \theta_2)$ then the previous calculations show that it is an improvement to remove the whole interval. In other words,

$$(\theta_1, \theta_2) \in \arg\max_{\theta_L, \theta_H} \Delta(\theta_H, \theta_L)$$
$$\text{s.t. } \theta_1 \leq \theta_L \leq \theta_H \leq \theta_2$$

This concludes the proof. *Q.E.D.*

Appendix

A Lemma on Derivative

Lemma. The partial of $\Delta(\theta_H, \theta_L)$ with respect to θ_H can be expressed as:

$$\frac{\partial \Delta}{\partial \theta_H}(\theta_H, \theta_L) = S(\theta_H; \theta^*) \frac{U'(c^*(\theta_H))}{\beta} \frac{\partial c^*(\theta_H)}{\partial \theta_H}$$

where $S(\theta; \theta^*)$ is defined by,

$$S(\theta, \theta^*) \equiv (y - \beta)(\theta - \theta^*)\theta^* f(\theta^*) - \int_{\theta^*}^{\theta} \left(\theta - \beta\tilde{\theta}\right) f\left(\tilde{\theta}\right) d\tilde{\theta}$$

Since $U'(c^*(\theta_H)) > 0$ and $\frac{\partial c^*(\theta_H)}{\partial \theta_H} > 0$, then $sign(\Delta_1) = sign(S(\theta_H, \theta^*))$.

Proof. We have

$$\begin{aligned}
\Delta_1(\theta_H, \theta_L) = {} & [\theta_H U(c^*(\theta_H)) + W(y - c^*(\theta_H))] f(\theta_H) \\
& - [\theta^*(\theta_H, \theta_L) U(c^*(\theta_H)) + W(y - c^*(\theta_H))] f(\theta^*) \frac{\partial \theta^*}{\partial \theta_H} \\
& + \int_{\theta^*(\theta_H, \theta_L)}^{\theta_H} \{\theta U'(c^*(\theta_H)) - W'(y - c^*(\theta_H))\} f(\theta) \frac{\partial c^*(\theta_H)}{\partial \theta_H} d\theta \\
& + \{\theta^*(\theta_H, \theta_L) U(c^*(\theta_L)) + W(y - c^*(\theta_L))\} f(\theta^*) \frac{\partial \theta^*}{\partial \theta_H} \\
& - [\theta_H U(c^*(\theta_H)) + W(y - c^*(\theta_H)) f(\theta_H)]
\end{aligned}$$

Combining terms,

$\Delta_1(\theta_H, \theta_L) =$

$$\begin{aligned}
& \left(\int_{\theta^*(\theta_H, \theta_L)}^{\theta_H} \{\theta U'(c^*(\theta_H)) - W'(y - c^*(\theta_H))\} f(\theta) d\theta\right) \frac{\partial c^*(\theta_H)}{\partial \theta_H} \\
& + \{\theta^*(\theta_H, \theta_L)[U(c^*(\theta_L)) - U(c^*(\theta_H))] + W(y - c^*(\theta_L)) - W(y - c^*(\theta_H))\} f(\theta^*) \frac{\partial \theta^*}{\partial \theta_H}
\end{aligned}$$

Now, from (18) we have

$$\theta U'[c^*(\theta)] - W'(y - c^*(\theta)) = \left[\frac{\beta - 1}{\beta}\right] \theta U'[c^*(\theta)]$$

Substituting above

$$\Delta_1(\theta_H,\theta_L) =$$

$$\left(\int_{\theta^*(\theta_H,\theta_L)}^{\theta_H}\left(\theta-\frac{1}{\beta}\theta_H\right)f(\theta)\,d\theta\right)U'(c^*(\theta_H))\frac{\partial c^*(\theta_H)}{\partial\theta_H}$$
$$+\left\{\theta^*(\theta_H,\theta_L)\left[U(c^*(\theta_L))-U(c^*(\theta_H))\right]+W(y-c^*(\theta_L))-W(y-c^*(\theta_H))\right\}f(\theta^*)\frac{\partial\theta^*}{\partial\theta_H}$$

We also have that from (17),

$$-\frac{\theta^*(\theta_H,\theta_L)}{\beta}\left[U(c^*(\theta_L))-U(c^*(\theta_H))\right]=\left\{W(y-c^*(\theta_L))-W(y-c^*(\theta_H))\right\}$$

So,

$$\begin{aligned}\Delta_1(\theta_H,\theta_L) &= \left\{\left[\frac{1}{\beta}-1\right]\theta^* f(\theta^*)\right\}\left[U(c^*(\theta_H))-U(c^*(\theta_L))\right]\frac{\partial\theta^*}{\partial\theta_H}\\ &\quad-\left(\int_{\theta^*}^{\theta_H}\left(\frac{1}{\beta}\theta_H-\theta\right)f(\theta)\,d\theta\right)U'(c^*(\theta_H))\frac{\partial c^*(\theta_H)}{\partial\theta_H}\end{aligned}$$

Differentiating (18) we obtain:

$$\frac{\partial\theta^*}{\partial\theta_H}\left[U(c^*(\theta_H))-U(c^*(\theta_L))\right]=-\left[\theta^*U'(c^*(\theta_H))-\beta W'(y-c^*(\theta_H))\right]\frac{\partial c^*(\theta_H)}{\partial\theta_H}$$

Using the fact that $\theta U'[c^*(\theta)]-\beta W'(1-c^*(\theta))=0$ this implies

$$\frac{\partial\theta^*}{\partial\theta_H}\left[U(c^*(\theta_H))-U(c^*(\theta_L))\right]=\left[\theta_H-\theta^*\right]U'\left[c^*(\theta_H)\right]\frac{\partial c^*(\theta_H)}{\partial\theta_H}$$

Substituting back the result follows. *Q.E.D.*

B Matlab burn.m Code: Money Burning with Three Types

```
function burn
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% computes potential money burning allocation for the 3 type case
% reports allocation and whether or not it satisfies
% auxiliary conditions to be deemed incentive compatible
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%% parameterization %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% sigma=2, thetal = .8, thetah=1.6, thetam=1, ploverph = .7 betta=.7
% leads to money burning for the middle type for high enough pm
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear
global betta sigma thetal thetah thetam lamda y p

% parameters
ploverph = .7;  %ratio of pl over ph

ppmm=(.001:.005:.999)';  %possible values for pm
sigma = 2;
thetal = .8;
thetah = 1.6;
thetam = 1;
betta = .7;
y=1;

% first best allocation
cl_fb = y*((thetal)^(-1/sigma)+1)^(-1);
ch_fb = y*((thetah)^(-1/sigma)+1)^(-1);
kl_fb = y - cl_fb;
kh_fb = y - ch_fb;
```

```
beta_star = thetal*(u(cl_fb) - u(ch_fb) ) / (u(kh_fb) - u(kl_fb));
% when betta<beta_star, first best for low and high not IC
Ym =0;  % dummy variable initialized

if (betta > thetal/thetam | betta < thetam/thetah )
        'no money burning possible-!!!! change parameters'
else

for i = 1: length(ppmm)
%    pause
    clc;
    pm=ppmm(i);

    %construction ph and pl given pm and the ratio pl/pm
    ph=(1-pm)/(1+ploverph);
    pl=1-pm-ph;

    % solve for mu
    A = [ -thetal/thetam, 1 ;  - betta , betta];
    B = [ -pm               ; -pm];
    mu=inv(A)*B;
    if mu(1) < 0 | mu(2) < 0;
        display('mu1 or mu2 is negative'); problem(i,1)=1;
    else

    % compute cL and cH from mu
    rl = thetal*(pl + mu(1))/(pl + betta*mu(1));
    % ratio of marginal utilities for the low type
    rh = thetah*(ph - mu(2)*thetam/thetah)/(ph - betta*mu(2) );
    % ratio of marginal utilities for the high type
    % if any of these ratios is negative .. this cannot be possible.. stop
    if rl<0 | rh<0; display('rl or rh are negative'); problem(i,1)=1.5; else

    % compute the allocation for low and high from the ratio
    % of marginal utilities and income
    c1l = y*[ 1 + rl.^(-1/sigma)].^-1; c2l = y - c1l;
```

```
c1h = y*[ 1 + rh.^(-1/sigma)].^-1; c2h = y - c1h;

u1l = u(c1l); u2l = u(c2l); u1h = u(c1h); u2h = u(c2h);

% now compute u1m and u2m from binding linear IC equations
% for low and medium types
Aic = [ thetal , betta ;  thetam , betta];
Bic = [ thetal*u1l + betta*u2l ; thetam*u1h + betta*u2h];

um = inv(Aic)*Bic;

%checking that those utility values are feasible
if (sigma-1)*um(1)>0 | (sigma-1)*um(2) >0
    ' utility is out of bounds'
    problem(i,1) = 2;
else

%finding the consumption bundle for the middle type
c1m =[(1-sigma)*um(1)].^(1/(1-sigma));
c2m =[(1-sigma)*um(2)].^(1/(1-sigma));
c1  = [c1l, c1m, c1h]
C1(i,:) = c1;

% check monotonicity, which is a necessary condition
% for IC and has not been imposed yet
if c1l > c1h | c1m > c1h | c1l > c1m | c2m > c2l | c2m < c2h | c2l < c2h
    'proposed solution is not IC!'
    problem(i,1)=3;
else

% check sign of lamdaH (multiplier of the resource for the high type)
% the multiplier for the low type is positive if mu is positive
if ph - thetam/thetah*mu(2) < 0 | ph - betta*mu(2) < 0 ;
    'lamdam turned out negative'
    problem(i,1) = 4;
else
```

```
    ym = c1m + c2m
    Ym(i,1) = ym;
    u1m = u(c1m); u2m = u(c2m);
    U = pl*[thetal*u1l + u2l] + pm*[thetam*u1m + u2m]+ ph*[thetah*u1h + u2h];
    Etheta = (pl*thetal+pm*thetam+ph*thetah);
    cpool = (1+Etheta^(-1/sigma))^(-1);
    Upool = Etheta*u(cpool)+u(1-cpool);

    if Upool > U
        'pooling is better :p'
        problem(i,1) = 5;
    else
        'separating is better than pooling'
    end
    end
    end
    end
    end
    end
    end
end

clf
figure(1)
titletext(1) = {'c1 allocation for l, m and h '};
titletext(2) = {['\sigma =',num2str(sigma),...
     ' \theta_l =',num2str(thetal),  '\theta_h =',num2str(thetam),...
     ' pl/ph =',num2str(ploverph), '\beta =',num2str(betta)]};
beta_star

plot(ppmm(find(problem==0)) , C1(find(problem==0),:))
grid
xlabel('pm')
ylabel('c_1')
legend('c_1_l','c_1_m','c_1_h')
```

```
title( titletext , 'fontweight' , 'bold')

figure(2)
plot(ppmm(find(problem==0)) , Ym(find(problem==0)))
grid
xlabel('p_m')
ylabel('c_m + k_m ')
title('Total consumption for m-type','fontweight','bold')

function f=u(x)
global sigma
f = (1/(1-sigma))*x.^(1-sigma);
```