Econometrica Supplementary Material

SUPPLEMENT TO "THE THEORY OF OPTIMAL DELEGATION WITH AN APPLICATION TO TARIFF CAPS"

(*Econometrica*, Vol. 81, No. 4, July 2013, 1541–1599)

BY MANUEL AMADOR AND KYLE BAGWELL

This supplement collects several results that are not to be published.

APPENDIX I: THE TRADE APPLICATION UNDER PERFECT COMPETITION

WE CONSIDER A STANDARD PERFECT COMPETITION MODEL of trade in three goods between two countries. Good x is imported by the home country and exported by the foreign country, while good y is exported by the home country and imported by the foreign country. The two countries also trade a numeraire good, n.

The demand side of the model is described as follows. We assume that residents in both countries share a common utility function. As is standard, this function is quasilinear and additively separable across the three products, where consumption of the numeraire good enters the utility function in a linear fashion. Within each country, consumer demand for good i, where $i = x, y$, is then a function of the local price of good i in relation to the price of good n. The supply side of the model is described by perfectly competitive markets. Hence, within each country, the production of good i, where $i = x, y$, is then also a function of the local price of good i in relation to the price of good n. In the standard manner, we assume further that the numeraire good is produced in each country under constant returns to scale, where the only factor is labor and the supply of labor is inelastic. We assume that the numeraire good is consumed in each country and freely traded across countries. Trade in the numeraire good then ensures that trade is balanced. The wage and price of the numeraire good are normalized to unity.

Under these assumptions, the market outcomes for goods x and y are independent. Without loss of generality, we may thus focus on good x. Under a symmetry assumption, whereby supply conditions take the mirror image form across the two countries, we may immediately represent foreign trade policies for good y once we represent home trade policies for good x. Hence, we now proceed with a two-good setting, in which the utility function for home consumers is given by $u(c^x) + c^n$, where c^x and c^n denote the respective consumption levels of goods x and n. Likewise, the utility function for foreign consumers is represented as $u(c_*^x) + c_*^n$, where the subscript $*$ indicates foreign-country variables. We assume that u is strictly increasing, strictly concave, and thrice continuously differentiable. We define p and p_*, respectively, as the home and foreign prices of good x. The home and foreign supply functions of good x are respectively given by $Q(p)$ and $Q_*(p_*)$. We assume that $Q(p)$ and

 DOI: 10.3982/ECTA9288

$Q_*(p_*)$ are strictly increasing and twice continuously differentiable for prices that generate strictly positive supply. We also assume that $Q(p) < Q_*(p)$ for any p such that there is strictly positive world supply. Thus, good x will be imported under free trade by the home country.[1]

We use z to represent the volume of trade across countries of good x. Home consumers' optimization delivers an inverse demand function for imports:

$$u'\big(Q(p)+z\big)=p \quad \Rightarrow \quad p=P(z), \tag{S.1}$$

where $P'(z)<0$ is confirmed below. Likewise, foreign consumers' optimization delivers an inverse supply function for exports:

$$u'\big(Q_*(p_*)-z\big)=p_* \quad \Rightarrow \quad p_*=P_*(z), \tag{S.2}$$

where $P'_*(z)>0$ is confirmed below.

We assume that each country employs a specific (i.e., per-unit) import tariff. Suppose now that the government of the home country selects the import tariff τ. The import tariff τ, in turn, implies an import volume, $z(\tau)$, where the implied import volume satisfies $\tau=P(z)-P_*(z)$. Given $P'_*(z)>0>P'(z)$, it follows that $z'(\tau)<0$. Of course, we may equivalently think of choosing the trade volume z, with an import tariff, $\tau(z)$, then implied by the relationship $\tau=P(z)-P_*(z)$, where, under our assumptions, $\tau'(z)<0$ follows.

Given a trade volume z, the home and foreign producer surplus (profit) functions are respectively defined as

$$\Pi(z)=\int_{\underline{p}}^{P(z)} Q(\tilde{p})\,d\tilde{p}, \quad \Pi_*(z)=\int_{\underline{p}_*}^{P_*(z)} Q_*(\tilde{p})\,d\tilde{p}.$$

In these expressions, $\underline{p}\geq 0$ and $\underline{p}_*\geq 0$ represent the highest prices below which supply is zero. We note that $\Pi(z)$ denotes the producer surplus enjoyed by the import-competing industry in the home country, while $\Pi_*(z)$ represents the producer surplus for the exporting industry in the foreign country.

We are now prepared to define government welfare functions. The welfare of the foreign government is constructed as the sum of the consumer surplus and the producer surplus in the foreign country:

$$V(z)=u\big(Q_*\big(P_*(z)\big)-z\big)-P_*(z)\big(Q_*\big(P_*(z)\big)-z\big)+\Pi_*(z). \tag{S.3}$$

The welfare of the domestic government is constructed in a similar fashion, except that it includes tariff revenue and also a political economy parameter reflecting the greater weight that the government may give to the interests of import-competing firms.[2] Formally, we represent the welfare of the domestic

[1] By symmetry, the home country exports good y under free trade.

[2] Recall that we put good y to the side. The welfare function of the foreign government includes tariff revenue and a political economy parameter for import-competing firms, when we consider good y.

government as

$$\begin{aligned}W(z|\gamma) &= u\big(Q\big(P(z)\big)+z\big)-P(z)\big(Q\big(P(z)\big)+z\big)\\&\quad+\big(P(z)-P_*(z)\big)z+\gamma\Pi(z), \quad \text{or}\\W(z|\gamma) &= B(z)+\gamma\Pi(z),\end{aligned}$$

where we let

$$B(z)\equiv u\big(Q\big(P(z)\big)+z\big)-P(z)Q\big(P(z)\big)-P_*(z)z, \tag{S.4}$$

and where $\gamma\in\Gamma\equiv[\underline{\gamma},\overline{\gamma}]$ is the domestic political economy parameter.[3]

Different specifications for the import tariff lead to different levels in the equilibrium volume of international trade, where the relationship between τ and z is one-to-one. For convenience, we use z as the policy variable and report the following result.

LEMMA S.1: *The following holds for all* $z>0$: $V'(z)>0$; $\Pi'(z)<0$; *and* $B''(z)+\Pi''(z)+V''(z)<0$.

PROOF: As a first step, we will show that $P'(z)<0$. To see this, note that taking the derivative with respect to z in equation (S.1) yields

$$u''\big(Q\big(P(z)\big)+z\big)\big(Q'\big(P(z)\big)P'(z)+1\big)=P'(z), \tag{S.5}$$

which solves to $P'(z)=-(Q'(P(z))-\frac{1}{u''(Q(P(z))+z)})^{-1}<0$.

We now show that $P'_*(z)>0$. To see this, note that taking the derivative with respect to z in (S.2) gives

$$u''\big(Q_*\big(P_*(z)\big)-z\big)\big(Q'_*\big(P_*(z)\big)P'_*(z)-1\big)=P'_*(z), \tag{S.6}$$

which solves to $P'_*(z)=(Q'_*(P_*(z))-\frac{1}{u''(Q_*(P_*(z))-z)})^{-1}>0$. Now note that $\Pi'(z)=Q(P(z))P'(z)<0$, and using equations (S.3) and (S.2), one finds that $V'(z)=zP'_*(z)>0$, where we used that $\Pi'_*(z)=Q_*(P_*(z))P'_*(z)$.

Using equation (S.4), together with (S.1), one finds that

$$\begin{aligned}B'(z) &= P(z)-P_*(z)-Q\big(P(z)\big)P'(z)-zP'_*(z),\\B''(z) &= P'(z)-2P'_*(z)-Q'\big(P(z)\big)\big(P'(z)\big)^2\\&\quad-Q\big(P(z)\big)P''(z)-zP''_*(z).\end{aligned}$$

[3] See Amador and Bagwell (2012) for consideration of the possibility that a government's private information concerns the value of tariff revenue.

From the definition of profits, we have that $\Pi''(z) = Q'(P(z))(P'(z))^2 + Q(P(z))P''(z)$. Using $V''(z) = zP_*''(z) + P_*'(z)$, we have that $B''(z) + \Pi''(z) + V''(z) = P'(z) - P_*'(z) < 0$. *Q.E.D.*

Intuitively, a higher volume of trade is delivered if the import tariff applied by the home government is lower. With $P_*'(z) > 0$, the foreign country enjoys a terms-of-trade gain. This explains why the welfare of the foreign country strictly increases as trade volume rises. Of course, with $P'(z) < 0$, a higher trade volume depresses the price of the imported good in the home market, leading to a strict decrease in the profit of the import-competing industry. Finally, with $\gamma = 1$, the joint welfare of the home and foreign government corresponds to global real income, which is maximized at the volume of trade that corresponds to free trade.

We also have the following result.

LEMMA S.2: *If $Q'' \le 0$, $Q_*'' \le 0$, and $u''' \ge 0$, then $V''(z) > 0$, $B''(z) < 0$, and $\Pi''(z) > 0$ for all $z > 0$.*

PROOF: Taking another derivative in equation (S.5), we obtain that

$$P''(z) = -\frac{Q''(P(z)) + \dfrac{u'''(Q(P(z)) + z)}{(u''(Q(P(z)) + z)^3}}{\left(Q'(P(z)) - \dfrac{1}{u''(Q(P(z)) + z)}\right)^3} \ge 0.$$

And taking another derivative in equation (S.6), we obtain that

$$P_*''(z) = -\frac{Q_*''(P_*(z)) + \dfrac{u'''(Q_*(P_*(z)) - z)}{(u''(Q_*(P_*(z)) - z))^3}}{\left(Q_*'(P_*(z)) - \dfrac{1}{u''(Q_*(P_*(z)) - z)}\right)^3} \ge 0.$$

Using our equation for $V''(z)$, it follows that $V''(z) = zP_*''(z) + P_*'(z) > 0$. Recall also the equation for $B''(z)$:

$$\begin{aligned} B''(z) &= P'(z) - 2P_*'(z) - Q'\big(P(z)\big)\big(P'(z)\big)^2 \\ &\quad - Q\big(P(z)\big)P''(z) - zP_*''(z) < 0. \end{aligned}$$

And finally, $\Pi''(z) = Q'(P(z))(P'(z))^2 + Q(P(z))P''(z) > 0$. *Q.E.D.*

Given that $\Pi'(z) < 0$, we may denote by Π^{-1} the inverse function of Π. Let $z(\pi) = \Pi^{-1}(\pi)$, $b(\pi) \equiv B(\Pi^{-1}(\pi))$, and $v(\pi) \equiv V(\Pi^{-1}(\pi))$. Let the set of feasible π be $[0, \Pi(0)] \equiv [0, \overline{\pi}]$. Using the findings above, we see that, for all

$\pi \in [0, \overline{\pi})$, $z'(\pi) < 0$ and $v'(\pi) < 0$. That is, a higher value for domestic profit corresponds to a lower trade volume and thus a higher import tariff; consequently, the welfare of the foreign government strictly falls when the domestic profit is increased. We can show further that a version of Lemma S.2 holds for these new functions: if $Q'' \leq 0$, $Q''_* \leq 0$, and $u''' \geq 0$, then $v''(\pi) > 0$ for all $\pi \in [0, \overline{\pi})$.[4] As we stated in the main text, we are thus careful not to exclude the possibility of a strictly convex foreign welfare function.

We now consider the linear-quadratic example. Recall that, in this example, $Q(p) = p/2$, $Q_*(p) = p$, and $u(c) = c - c^2/2$. The political shocks are distributed over $[\underline{\gamma}, \overline{\gamma}]$, where $1 \leq \underline{\gamma} < \overline{\gamma} < 7/4$. Note as well that, in this example, a tariff higher than $1/6$ is prohibitive.[5]

When trade volume z is treated as the independent variable, we find that $P(z) = \frac{2}{3}(1 - z)$, $P_*(z) = \frac{1}{2}(1 + z)$, $\Pi(z) = (1 - z)^2/9$, $\Pi_*(z) = (1 + z)^2/8$, $V(z) = \frac{1}{4}(1 + z^2)$, and $B(z) = \frac{1}{18}(1 + 7z - 17z^2)$. Letting π denote domestic profits as before, we can rewrite these functions in terms of π:

$$b(\pi) = \frac{1}{2}(-1 + 9\sqrt{\pi} - 17\pi), \quad v(\pi) = \frac{1}{4}(2 - 6\sqrt{\pi} + 9\pi).$$

APPENDIX J: ALONSO AND MATOUSCHEK (2008) AS A SPECIAL CASE

The following is Alonso and Matouschek's (2008) main result regarding the optimality of what they called *threshold delegation*.

PROPOSITION AM: *Threshold delegation is optimal if and only if there exists* $\gamma_L, \gamma_H \in (\underline{\gamma}, \overline{\gamma})$ *such that* $\gamma_H > \gamma_L$ *and* (AMi) $S(\gamma_H) = 0$ *and* $S(\gamma) \geq 0$ *for* $\gamma > \gamma_H$; (AMii) $T(\gamma_L) = 0$ *and* $T(\gamma) \leq 0$ *for* $\gamma < \gamma_L$; (AMiii) T *convex in* $[\gamma_L, \gamma_H]$; *where*:

$$T(\gamma) = F(\gamma)\big(\gamma - \mathbb{E}\big[\pi_P(z)|z \leq \gamma\big]\big),$$
$$S(\gamma) = \big(1 - F(\gamma)\big)\big(\gamma - \mathbb{E}\big[\pi_P(z)|z \geq \gamma\big]\big).$$

Note that threshold delegation as defined in Alonso and Matouschek (2008) requires both γ_H and γ_L to be interior. We now show that the conditions above are equivalent to the conditions in part (a) of our Propositions 1 and 2, that is, to conditions (c1), (c2), (c2′), (c3), and (c3′), with κ as given by equation (2). Given that γ_L and γ_H are interior, the conditions that are relevant from part (a) of Propositions 1 and 2 are (c1), (c2), and (c3).

[4]This follows directly from using that $z''(\pi) = -\frac{\Pi'' z'}{(\Pi')^2} > 0$ from Lemma S.2. Then it follows that $v''(\pi) = V''(z'^2) + V'z''$. Using that $V'' > 0$ from Lemma S.2 and that $V' > 0$, the result follows.

[5]A government with a political pressure γ higher than or equal to $7/4$ prefers a prohibitive tariff.

Then we have the following equivalence results.

Condition (AMi) *is equivalent to* (c2) *of part* (a) *of Propositions* 1 *and* 2. Note that $S(\gamma)$ can be written as

$$S(\gamma) = \int_{\gamma}^{\overline{\gamma}} \big(\gamma - \pi_P(\tilde{\gamma})\big) f(\tilde{\gamma})\, d\tilde{\gamma}.$$

Now note that the inequality in condition (c2) of part (a) of Propositions 1 and 2 can be written equivalently here as

$$\begin{aligned} -\int_{\gamma}^{\overline{\gamma}} \big(\gamma_H - \pi_p(\tilde{\gamma})\big) f(\tilde{\gamma})\, d\tilde{\gamma} &\le (\gamma - \gamma_H)\big(1 - F(\gamma)\big) \\ \iff \quad \int_{\gamma}^{\overline{\gamma}} \big[\big(\gamma_H - \pi_p(\tilde{\gamma})\big) + (\gamma - \gamma_H)\big] f(\tilde{\gamma})\, d\tilde{\gamma} &\ge 0 \\ \iff \quad \int_{\gamma}^{\overline{\gamma}} \big(\gamma - \pi_p(\tilde{\gamma})\big) f(\tilde{\gamma})\, d\tilde{\gamma} &\ge 0. \end{aligned}$$

And thus condition (c2) is equivalent to requiring $S(\gamma) \ge 0$ for all $\gamma \ge \gamma_H$, with equality at γ_H. Hence (c2) of part (a) of Propositions 1 and 2 is equivalent to (AMi).

Condition (AMii) *is equivalent to* (c3) *of part* (a) *of Propositions* 1 *and* 2. The proof mirrors the proof for condition (AMi) above, so we omit it.

Condition (AMiii) *is equivalent to* (c1) *of part* (a) *of Propositions* 1 *and* 2. Condition (AMiii) is that $T(\gamma)$ be convex. Note that, from the definition of T, we have

$$\begin{aligned} T(\gamma) &= F(\gamma)\big(\gamma - \mathbb{E}\big[\pi_P(z) | z \le \gamma\big]\big), \\ T'(\gamma) &= \gamma f(\gamma) + F(\gamma) - \pi_P(\gamma) f(\gamma) \\ &= F(\gamma) + \big(\gamma - \pi_P(\gamma)\big) f(\gamma). \end{aligned}$$

Convexity of T requires then that $F(\gamma) + (\gamma - \pi_P(\gamma)) f(\gamma)$ be non-decreasing for $\gamma \in [\gamma_L, \gamma_H]$. Using that $w_\pi(\gamma, \pi_f(\gamma)) = -(\gamma - \pi_p(\gamma))$, it follows that this requirement is equivalent to condition (c1) of part (a) of Propositions 1 and 2.

So the conditions in Alonso and Matouschek (2008) are the same as the conditions of part (a) of Propositions 1 and 2.[6]

[6]Alonso and Matouschek (2008) also obtained a result regarding upper-threshold delegation that applies when $\gamma_L = \underline{\gamma}$. This is a case that is also covered by part (a) of Propositions 1 and 2.

APPENDIX K: AMBRUS AND EGOROV: GENERALIZATION

The Mapping From Ambrus and Egorov to Our Framework

We first confirm the mapping from the quadratic-uniform model analyzed by Ambrus and Egorov (2009) to our modeling framework. Ambrus and Egorov (2009) specified a program in the form of (P2), where the integrand of the objective function is

$$\widetilde{w}(\gamma, \pi) - \widetilde{t}(\gamma) \equiv -\left[\alpha(\pi-\gamma)^2 + (\pi-\gamma-\beta)^2\right] - \widetilde{t}(\gamma),$$

while the agent maximizes

$$\widetilde{u}(\gamma, \pi) - \widetilde{t}(\gamma) \equiv -\left[\pi^2 - 2\pi(\gamma+\beta) + (\gamma+\beta)^2\right] - \widetilde{t}(\gamma),$$

where $\widetilde{t}(\gamma)$ is a money burning variable. Dividing by 2, ignoring constants, and defining $t(\gamma) \equiv \widetilde{t}(\gamma)/2$, we can represent their framework as specifying that the integrand of the objective function is

$$w(\gamma, \pi) - t(\gamma) = -\frac{\alpha+1}{2}\left(\pi - \gamma - \frac{\beta}{\alpha+1}\right)^2 - t(\gamma),$$

while the agent maximizes

$$u(\gamma, \pi) - t(\gamma) = b(\pi) + \pi\gamma - t(\gamma),$$

with $b(\pi) \equiv \beta\pi - \frac{\pi^2}{2}$.

More General Distribution Functions

We establish here the following corollary.

COROLLARY S.1: *Consider the generalized Ambrus and Egorov (2009) quadratic model with $\alpha \leq 1$. If* (i) $\mathbb{E}[\gamma] - \underline{\gamma} > \frac{\alpha\beta}{\alpha+1}$, *then there exists $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that $\mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = \alpha\beta/(1+\alpha)$. If, in addition,* (ii) $F(\gamma) + \alpha\beta f(\gamma)$ *is nondecreasing for $\gamma \in [\underline{\gamma}, \gamma_H]$, and* (iii) $(1+\alpha)\mathbb{E}[\widetilde{\gamma}|\widetilde{\gamma} \geq \gamma] - \gamma \leq \alpha(\gamma_H + \beta)$ *for $\gamma \in [\gamma_H, \overline{\gamma}]$, then a cap allocation with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ is optimal.*

PROOF: To establish that a cap allocation with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ is optimal when money burning is allowed, we refer to part (b) of Proposition 1. We see that we need to find $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that conditions (c1), (c2), and (c3′) with κ defined by (3) are satisfied when $\gamma_L = \underline{\gamma}$. Note that condition (c3′) is satisfied, since $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) = -\alpha\beta < 0$.

We may express condition (c2) with κ defined by (3) as

$$(1+\alpha)\mathbb{E}(\tilde{\gamma}|\tilde{\gamma} \geq \gamma) - \alpha(\gamma_H + \beta) - \gamma \leq 0, \quad \forall \gamma \in [\gamma_H, \overline{\gamma}], \tag{S.7}$$

with equality at $\gamma = \gamma_H$. Let $q(\gamma, \gamma_H) \equiv (1+\alpha)\mathbb{E}(\tilde{\gamma}|\tilde{\gamma} \geq \gamma) - \alpha(\gamma_H + \beta) - \gamma$. Note that $q(\gamma, \gamma)$ is continuous in γ; equals $(1+\alpha)(\mathbb{E}[\gamma] - \underline{\gamma}) - \alpha\beta$ at $\underline{\gamma}$, which is strictly positive by hypothesis (i); and equals $-\alpha\beta$ at $\overline{\gamma}$, which is negative. Then there exists a $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that $q(\gamma_H, \gamma_H) = 0$, or equivalently such that $\mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = \alpha\beta/(1+\alpha)$. Hypothesis (iii) then implies that (S.7) holds for all $\gamma \in [\gamma_H, \overline{\gamma}]$.

Condition (c1) with κ defined by (3) holds if $F(\gamma) + \alpha\beta f(\gamma)$ is nondecreasing for all $\gamma \in [\underline{\gamma}, \gamma_H]$ which is true by hypothesis (ii). We may now apply part (b) of Proposition 1. *Q.E.D.*

Recall the definition of $d(\gamma)$ from the main text: $d(\gamma) = \mathbb{E}[\tilde{\gamma}|\tilde{\gamma} > \gamma] - \gamma$. Now we show that if d is convex, then hypothesis (iii) of Corollary S.1 holds. But before doing this, let us first state some useful properties of the function $d(\gamma)$.

LEMMA S.3: *The function $d(\gamma)$ satisfies the following properties*: (i) *$d(\gamma)$ is continuous and differentiable*; *and* (ii) $d'(\overline{\gamma}) = -1/2$.

PROOF: Part (i) follows directly from the definition of $d(\gamma)$ and the continuity of f. Note that taking the derivative of d, we get

$$d'(\gamma) = -1 + d(\gamma)\frac{f(\gamma)}{1 - F(\gamma)},$$

which is well defined for all $\gamma < \overline{\gamma}$. Part (ii) then follows from the limit of this equation as $\gamma \to \overline{\gamma}$ and using L'Hôpital's rule. *Q.E.D.*

Under hypothesis (i), we have already shown that equation (S.7) holds at γ_H. We may thus rewrite equation (S.7) once more as

$$\frac{d(\gamma) - d(\gamma_H)}{\gamma - \gamma_H} \leq \frac{-\alpha}{\alpha + 1} \quad \text{for all } \gamma \in (\gamma_H, \overline{\gamma}].$$

If d is convex for $\gamma \in [\gamma_H, \overline{\gamma}]$, then we have that

$$\frac{d(\gamma) - d(\gamma_H)}{\gamma - \gamma_H} \leq d'(\overline{\gamma}) = -\frac{1}{2} \quad \text{for all } \gamma \in [\gamma_H, \overline{\gamma}].$$

Therefore, since $\alpha \leq 1$ ensures that $-1/2 \leq -\alpha/(\alpha + 1)$, we conclude that equation (S.7) holds. And hence, hypothesis (iii) of Corollary S.1 holds if d is convex.

Other Results Concerning the Ambrus and Egorov Generalization

The next lemma collects some other useful properties of $d(\gamma)$.

LEMMA S.4: *Let $f(\gamma)$ be differentiable. The function $d(\gamma)$ satisfies the following properties*:

(i) $d(\gamma) > 0$ *for all* $\gamma \in [\underline{\gamma}, \overline{\gamma})$.

(ii) $d'(\gamma) = -1 + d(\gamma)\frac{f(\gamma)}{1-F(\gamma)}$ *for all* $\gamma \in [\underline{\gamma}, \overline{\gamma})$.

(iii) $d(\underline{\gamma}) = E(\gamma) - \underline{\gamma} > 0 = d(\overline{\gamma})$.

(iv) *If* $\frac{d}{d\gamma}(\frac{1-F(\gamma)}{f(\gamma)}) < 0$ *for all* $\gamma \in \Gamma$, *then* $d'(\gamma) < 0$ *for all* $\gamma \in [\underline{\gamma}, \overline{\gamma})$.

(v) $d'(\overline{\gamma}) = -1/2$.

(vi) *If* $f'(\gamma) \geq 0$ *for all* $\gamma \in \Gamma$, *then* $d(\gamma) \leq \frac{1-F(\gamma)}{2f(\gamma)}$ *for all* $\gamma \in \Gamma$.

PROOF: Recall that $d(\gamma)$ is continuous and differentiable by Lemma S.3. We consider the various properties in sequence:

(i) After integrating by parts, we get

$$d(\gamma) = \frac{\int_{\gamma}^{\overline{\gamma}} [1 - F(\tilde{\gamma})]\, d\tilde{\gamma}}{1 - F(\gamma)} > 0 \quad \text{for all } \gamma < \overline{\gamma}. \tag{S.8}$$

(ii) This follows immediately after differentiating (S.8).

(iii) We may use the definition of $d(\gamma)$ to observe that $d(\underline{\gamma}) = E(\gamma) - \underline{\gamma} > 0$. Given $f(\overline{\gamma}) > 0$, we may use (S.8) and L'Hôpital's rule to confirm that $d(\overline{\gamma}) = 0$.

(iv) Suppose $d'(\gamma) = 0$ at some point $\widehat{\gamma} \in [\underline{\gamma}, \overline{\gamma})$. Then by Lemma S.4, part (ii), we have that $d(\widehat{\gamma}) = \frac{1-F(\widehat{\gamma})}{f(\widehat{\gamma})}$. Hence, $d''(\widehat{\gamma}) = \frac{1-F(\widehat{\gamma})}{f(\widehat{\gamma})} \frac{d}{d\gamma}[\frac{f(\gamma)}{1-F(\gamma)}]|_{\gamma=\widehat{\gamma}} > 0$, where the inequality uses the assumption that $\frac{d}{d\gamma}(\frac{1-F(\gamma)}{f(\gamma)}) < 0$ for all $\gamma \in \Gamma$. We conclude that an extremum of $d(\gamma)$ in the domain $[\underline{\gamma}, \overline{\gamma})$ must be a local minimum. This implies, in turn, that the supposition leads to a contradiction. Since $d(\gamma) > 0 = d(\overline{\gamma})$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma})$, if the function $d(\gamma)$ achieves a local minimum at $\widehat{\gamma} \in [\underline{\gamma}, \overline{\gamma})$, then it must achieve a local maximum at some "turnaround" point $\gamma^* \in (\widehat{\gamma}, \overline{\gamma})$, which contradicts the conclusion just obtained. It follows that $d'(\gamma) < 0$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma})$. We establish that $d'(\overline{\gamma}) < 0$ in the proof of part (v).

(v) This follows from Lemma S.3.

(vi) This follows from Lemma 9. *Q.E.D.*

We now establish some results about the second derivative of the function $d(\gamma)$.

LEMMA S.5: *Let $f(\gamma)$ be differentiable. The function $d(\gamma)$ satisfies the following properties*:

(i) $d''(\gamma) = -\frac{f(\gamma)}{1-F(\gamma)} + d(\gamma)[\frac{(1-F(\gamma))f'(\gamma)+2f^2(\gamma)}{(1-F(\gamma))^2}]$ *for all* $\gamma \in [\underline{\gamma}, \overline{\gamma})$.
(ii) $d''(\overline{\gamma}) = \frac{f'(\overline{\gamma})}{6f(\overline{\gamma})}$.
(iii) $d''(\underline{\gamma}) = -f(\underline{\gamma}) + [E(\gamma) - \underline{\gamma}][f'(\underline{\gamma}) + 2f^2(\underline{\gamma})]$.
(iv) *If* $f'(\gamma) \geq 0$ *for all* $\gamma \in \Gamma$, *then* $d''(\gamma) \leq \frac{1}{2}\frac{f'(\gamma)}{f(\gamma)}$ *for all* $\gamma \in \Gamma$.

PROOF:
(i) This result follows directly after using Lemma S.4, part (ii).
(ii) Using (S.8) and part (i) of Lemma S.5, this result follows after three applications of L'Hôpital's rule, in order to take the limit of $d''(\gamma)$ as $\gamma \to \overline{\gamma}$.
(iii) This follows easily from part (i) of Lemma S.5, after using part (iii) of Lemma S.4.
(iv) Using Lemma S.5, part (i), and Lemma S.4, part (vi), we find that, if $f'(\gamma) \geq 0$ for all $\gamma \in \Gamma$, then

$$\begin{aligned} d''(\gamma) &\leq -\frac{f(\gamma)}{1-F(\gamma)} + \frac{1}{2f(\gamma)}\left[\frac{(1-F(\gamma))f'(\gamma) + 2f^2(\gamma)}{1-F(\gamma)}\right] \\ &= \frac{f'(\gamma)}{2f(\gamma)} \quad \text{for all } \gamma \in \Gamma. \end{aligned}$$

Q.E.D.

We now identify sufficient conditions for the convexity of $d(\gamma)$.

LEMMA S.6: *Assume* $f(\gamma)$ *is twice differentiable and*
(i) $f'(\gamma) \geq 0$ *for all* $\gamma \in \Gamma$,
(ii) *if there exists* $\gamma \in (\underline{\gamma}, \overline{\gamma})$ *such that* $f'(\gamma) > 0$, *then* $f'(\overline{\gamma}) > 0$,
(iii) $f''(\gamma) \leq [2(f'(\gamma))^2/f(\gamma)] + [f'(\gamma)f(\gamma)/(1-F(\gamma))]$, *for all* $\gamma \in [\underline{\gamma}, \overline{\gamma})$.
Then $d''(\gamma) \geq 0$ *for all* $\gamma \in \Gamma$.

PROOF: If the distribution were uniform, then all of the assumptions in Lemma S.6 clearly hold. In fact, in the case of a uniform distribution, we have that $d'(\gamma) \equiv 1/2$, and so $d''(\gamma) = 0$. Suppose, henceforth, that the distribution is not uniform.

Suppose that there exists some $\widehat{\gamma} \in (\underline{\gamma}, \overline{\gamma})$ such that $d'(\widehat{\gamma}) > d'(\overline{\gamma})$. By Lemma S.6, parts (i) and (ii), we thus must have that $f'(\overline{\gamma}) > 0$, and so $d''(\overline{\gamma}) > 0$ follows from part (ii) of Lemma S.5. This means that there must exist $\gamma^* \in (\widehat{\gamma}, \overline{\gamma})$ such that $d''(\gamma^*) = 0$ and $d'''(\gamma^*) > 0$. But this contradicts parts (i) and (iii) of Lemma S.6, as we now show.

Specifically, using part (i) of Lemma S.5, we find that $d'' = 0$ implies

$$d(\gamma) = \frac{f(\gamma)(1-F(\gamma))}{(1-F(\gamma))f'(\gamma) + 2f^2(\gamma)}.$$

Next, using part (i) of Lemma S.5 and also part (ii) of Lemma S.4, and after some simplification, we get

$$\begin{aligned} d'''(\gamma) &\\ &= -\frac{2(1-F(\gamma))f'(\gamma)+3f^2(\gamma)}{(1-F(\gamma))^2} \\ &\quad + d(\gamma)\left\{\frac{6(1-F(\gamma))f'(\gamma)f(\gamma)+6f^3(\gamma)+(1-F(\gamma))^2 f''(\gamma)}{(1-F(\gamma))^3}\right\}. \end{aligned}$$

So, if $d''=0$, then we may use the value for $d(\gamma)$ derived just above and, after some simplification, determine that

$$\begin{aligned} d'''(\gamma) = \{&-f'(\gamma)\big[\big(1-F(\gamma)\big)f'(\gamma)+f^2(\gamma)\big] \\ &+\big(1-F(\gamma)\big)\big[f(\gamma)f''(\gamma)-\big(f'(\gamma)\big)^2\big]\} \\ &/\{\big(1-F(\gamma)\big)\big[\big(1-F(\gamma)\big)f'(\gamma)+2f^2(\gamma)\big]\}. \end{aligned}$$

Now, the denominator is positive provided that $f(\gamma)/[1-F(\gamma)]$ is increasing; in particular, the denominator is strictly positive under part (i) of Lemma S.6. Thus, $d''' \leq 0$ when $d''=0$ is true if and only if

$$f''(\gamma) \leq \frac{2(f'(\gamma))^2}{f(\gamma)} + \frac{f'(\gamma)f(\gamma)}{(1-F(\gamma))},$$

which is part (iii) of Lemma S.6. Thus, under part (iii) of Lemma S.6, we cannot have $d''(\gamma^*)=0$ and $d'''(\gamma^*)>0$.

It thus must be that $d'(\gamma) \leq d'(\overline{\gamma})$ for all $\gamma \in \Gamma$. Now suppose that $d''(\widehat{\gamma})<0$ at some $\widehat{\gamma} \in [\underline{\gamma}, \overline{\gamma})$. Then, for $\epsilon > 0$ sufficiently small, $d'(\widehat{\gamma}+\epsilon) < d'(\overline{\gamma})$ and $d''(\widehat{\gamma}+\epsilon)<0$. This means that there must be some "turn-around" point, $\gamma^* \in (\widehat{\gamma}+\epsilon, \overline{\gamma})$, at which $d'(\gamma)$ hits a local minimum and begins to climb up to $d'(\overline{\gamma})$. Thus, $d''(\gamma^*)=0<d'''(\gamma^*)$. But this contradicts part (iii) of Lemma S.6.

Thus, the only remaining option is that $d'(\gamma) \leq d'(\overline{\gamma})$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ and $d''(\gamma) \geq 0$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. *Q.E.D.*

And finally, we apply the results to the power distribution.

LEMMA S.7: *For the power distribution with $n \geq 1$, $d''(\gamma) \geq 0$.*

PROOF: With the power distribution, $f(\gamma)=n\gamma^{n-1}$, $F(\gamma)=\gamma^n$, and $\underline{\gamma}=0<1=\overline{\gamma}$. Consider the parts of Lemma S.6 in sequence:

(i) $f'(\gamma) \geq 0$, for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This condition clearly holds for $n \geq 1$.

(ii) If there exists $\gamma \in [\underline{\gamma}, \overline{\gamma})$ such that $f'(\gamma)>0$, then $f'(\overline{\gamma})>0$. Under the supposition, we must have $n>1$, and in this case it follows that $f'(\overline{\gamma})>0$.

(iii) $f''(\gamma) \leq [2(f'(\gamma))^2/f(\gamma)] + [f'(\gamma)f(\gamma)/(1 - F(\gamma))]$, for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This condition holds for $n = 1$ trivially. For $n > 1$, straightforward calculations confirm that $f''(\gamma) \leq [2(f'(\gamma))^2/f(\gamma)]$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, and so the condition holds easily (we do not even use the second and positive term on the right hand side). *Q.E.D.*

REFERENCES

ALONSO, R., AND N. MATOUSCHEK (2008): "Optimal Delegation," *The Review of Economic Studies*, 75 (1), 259–293. [5,6]

AMADOR, M., AND K. BAGWELL (2012): "Tariff Revenue and Tariff Caps," *American Economic Review, Papers and Proceedings*, 102 (3), 459–465. [3]

AMBRUS, A., AND G. EGOROV (2009): "Delegation and Nonmonetary Incentives," Working Paper, Harvard University. [7]

Dept. of Economics, Stanford University, Stanford, CA 94305, U.S.A.; amador@stanford.edu

and

Dept. of Economics, Stanford University, Stanford, CA 94305, U.S.A.; kbagwell@stanford.edu.

Manuscript received May, 2010; final revision received June, 2012.