Learning from Prices: Public Communication and Welfare

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We study the effect of releasing public information about productivity or monetary shocks using a micro-founded macroeconomic model in which agents learn from the distribution of nominal prices. While a public release has the direct beneficial effect of providing new information, it also has the indirect adverse effect of reducing the informational efficiency of the price system. We show that the negative indirect effect can dominate. Thus, the public information release may increase uncertainty about the monetary shock and reduce welfare. We find that the optimal communication policy is always to release either all or none of the information.

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I. Introduction

Economic statistics are noisy. For example, the first estimates of GDP in the United States, published by the Bureau of Economic Analysis, are very imprecise. Only half the data required to compute GDP are known completely at the time of the first release, and the first numbers are subsequently subject to significant revisions.[1] Should one be concerned that releasing noisy statistics may create confusion and lead the private sector to act on incorrect information? At first pass, the notion that more public information leads to more uncertainty, and thus to worse decisions, is unwarranted. Indeed, if we treat other sources of information as exogenous, then a rational Bayesian decision maker will always be better informed after observing a public signal, however noisy. But in reality, not all sources of information are exogenous. Households and firms learn through their interactions in markets and from observing endogenous variables such as prices. In this paper, we show that the release of public signals about aggregate fundamentals can make such endogenous sources of information less precise, leading to more confusion and greater uncertainty than no release at all.

To study the effect of public information releases, we propose a version of Lucas's (1972) island model based on Woodford's (2003) specification of preferences and technology. Our baseline economy is inhabited by a representative family of competitive workers who supply differentiated intermediate goods. The economy is subject to shocks that workers cannot observe directly: an aggregate monetary shock; an aggregate productivity shock; and idiosyncratic real demand shocks, one for each intermediate good market. Workers use all available information, including that revealed by prices, to forecast their respective real demand shocks and to make optimal labor supply decisions. As in Lucas's study, workers cannot tell whether a high nominal price in their own sector is due to a high real demand shock or to a high monetary shock.[2] In contrast to Lucas's study, though, workers learn about the monetary shock by observing the aggregate price level. However, the price level remains only partially revealing about the monetary shock because it is also affected by the unknown productivity shock.

In our model, the informativeness of the aggregate price level about the monetary shock is endogenous. Indeed, workers use the aggregate price level as a signal to improve their forecast of the monetary shock, which, along with other information, helps them decide on how much labor to supply. However, through market clearing, the aggregate labor

[1] See, e.g., "Why America's Advance GDP Figures Do Not Paint the Whole Picture," *Economist*, January 31, 2008.

[2] We show that this makes monetary shocks expansionary and amplifies the impact of productivity shocks.

supply feeds back into the price level and ultimately determines its informativeness.

To understand the mechanics of this feedback, it is helpful to think of workers forecasting the monetary shock in two steps. At first, they form a *public* forecast about the monetary shock using only publicly available information to filter out the productivity shock from the price level. Second, they each form a *private* forecast about the monetary shock, using their respective private information about productivity. The labor supply decision of a worker results from the weighted combination of these two forecasts. Note that because the public forecast is the same for everyone, all there is to learn about monetary shocks comes from other workers' private forecasts. Thus, what determines the informativeness of the price level is the weight that workers collectively assign to their private forecasts.

Because of this feedback, workers' weighting decisions can become strategic complements. Indeed, suppose that all workers were forced to put more weight on their private forecasts. As a result, more private information would feed into prices, making the price level more informative. This, in turn, would improve the quality of workers' forecasts. Crucially, in some cases, a worker's private forecast improves by more than the public forecast. This creates the strategic complementarity: a worker's optimal response is to follow others and to put more weight on her private forecast.

What, then, is the effect of releasing partial information about the monetary shock, the productivity shock, or both at once? As we stated at the beginning, everything else equal, such releases have the direct beneficial effect of providing new public information, which improves the quality of workers' public forecasts. But there is a countervailing equilibrium effect: workers put more weight on their public forecast, now of higher quality, and less on their private forecasts. This change in behavior reduces the endogenous informational content of prices. The strategic complementarities discussed earlier amplify the initial negative effect: households put less weight on their private forecasts, making prices less informative. This prompts households to put even less weight on their private forecasts, making prices even less informative, and so on. In fact, because of this amplification mechanism, the negative effect can dominate the positive effect in equilibrium, increasing households' uncertainty about the monetary shock and their real demands and reducing welfare. The amplification is necessary for the result: without it, we show that public information is always beneficial.

One important parameter in our analysis is the elasticity of labor supply.[3] This elasticity ultimately determines whether the information

[3] While in our basic model the micro and the macro elasticities are the same, we propose

content of prices is determined predominantly by the intermediate good demand (which arises from fully informed final good firms) or by the labor supply of imperfectly informed intermediate goods workers. We show that if this elasticity is high, then prices mainly aggregate the dispersed information of workers, reinforcing the role of strategic complementarities.

The cornerstone of our analysis is a learning externality that has been studied before in the work of Vives (1993), Morris and Shin (2005), and Amato and Shin (2006). These early papers do not feature the strategic complementarities necessary for our negative welfare results. More recently, however, Ganguli and Yang (2009) have independently studied complementarities similar to ours, but they focus on their implications for information acquisition in noisy-rational financial markets. To study the effects of learning externalities on the social value of public information, we develop a micro-founded macroeconomic model building on the preference and technology specification used by Woodford (2003), which has been recently extended to dispersed-information settings (see, e.g., Angeletos and La'O 2008; Hellwig and Venkateswaran 2009). Our results are not, however, tied to the specifics of this model: similar welfare results would also arise in an island model with cash-in-advance constraints.[4]

Morris and Shin (2002) have proposed a different mechanism for generating welfare-reducing public information releases: because of externalities in their payoffs, agents suffer from a socially harmful desire to coordinate. In later work, Hellwig (2005), Roca (2006), and Lorenzoni (2010) study the implications of Morris and Shin's findings within neo-Keynesian frameworks. Using linear-quadratic preferences, Angeletos and Pavan (2007) find conditions under which public information releases are welfare reducing and show that these negative welfare results are sensitive to the particular kind of externality assumed in the payoff structure. In contrast with this line of work, our results are not driven by any form of payoff externality. We show that our baseline model admits a form of payoff separation across sectors: workers in one sector do not directly care about the actions taken by others. Instead, our results are generated from an information externality that makes public information releases welfare reducing by increasing agents' uncertainty about fundamentals.[5] Our model also has a different

in online App. B an extension in which the two are allowed to differ. In this context, we show that the crucial parameter for our result is not the micro but the macro elasticity.

[4] See the working paper version of this paper (Amador and Weill 2008) for details. Although their work is based on a specification with different information and market structures, Angeletos and La'O (2008) have shown how appropriately designed Pigouvian taxes can correct the learning externality and overturn our negative welfare results.

[5] In Sec. VII.B, we extend the model and show that our results are robust to the addition of socially beneficial coordination motives.

positive implication: the publication of economic statistics can result in less accurate forecasts.

Also related to our paper is the recent work on global games, especially Angeletos and Werning (2006) and Hellwig, Mukherji, and Tsyvinski (2006), which introduce learning from others and study its impact on equilibrium selection. Finally, several authors have studied the interactions of public communication with public policy (see Taub 1997; Atkeson, Chari, and Kehoe 2007; Moscarini 2007; Eusepi and Preston 2010).

The rest of this paper is organized as follows. Section II sets up the basic model, Section III defines equilibrium, and Section IV characterizes the equilibrium set. Section V contains our main results concerning the welfare effect of public announcements. Section VI discusses the reasons for the negative welfare results. Section VII presents two robustness checks and extensions to the model. Section VIII presents conclusions. All proofs are collected in Appendix A.

II. The Model

We consider a standard money-in-the-utility-function model extended to incorporate three features of interest. First, the economy is affected by random productivity and nominal shocks. Second, both of these types of shocks are imperfectly and differentially known by agents in the economy. Finally, agents observe all nominal prices in the economy and learn from them. Time is discrete, and although the analysis of the model will be essentially static, we let time be infinite so that money is valued.

A. *Preferences and Technology*

There is a representative family composed of a [0, 1] continuum of workers who produce intermediate goods, indexed by i, a final good producer, and a shopper.[6] The utility of the family is

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t+\frac{1}{V}\log\frac{M_{t-1}^d}{P_t}-\frac{\delta}{1+\delta}\int_0^1 L_{it}^{1+1/\delta}di\right)\right],$$

where $\beta\in(0,1)$ is the discount factor and $\delta\geq 0$ is the Frisch elasticity of labor supply. In the utility function, C_t is the family's aggregate consumption of final goods, M_{t-1}^d is the money balance acquired in period

[6] The assumption of a large family has a long history in monetary models; see, e.g., the discussion in Woodford (2003, 144–45). See Lucas (1980) for an early statement of the shopper-worker metaphor in a monetary model.

$t-1$,[7] P_t is the price of the final good, and L_{it} is the hours worked by worker i. The budget constraint of the household in every period is

$$C_t + \frac{M_t^d}{P_t} \leq \int_0^1 \frac{P_{it}}{P_t}\Theta_i L_{it} di + \frac{M_{t-1}^d}{P_t} + \frac{\Pi_t}{P_t},$$

where, during period t, P_{it} is the nominal price of intermediate good $i \in [0, 1]$, Π_t are the nominal profits of the final good producers, and Θ_i is a productivity shock discussed below.

Each of the workers specializes in producing a differentiated intermediate good, also indexed by $i \in [0, 1]$. The production function of this intermediate is linear: L_i hours worked generate $Y_i = \Theta_i L_i$ units of intermediate good i. Intermediate goods are subsequently sold to final goods producers who assemble them using the Cobb-Douglas technology:

$$Y_t = \prod_{i \in [0,1]} Y_{it}^{A_i}, \tag{1}$$

where Y_t is the amount of the final good produced. The value A_i indexes the share of the intermediate good i in final good production.[8] We assume that the technology has constant returns to scale, and thus $\int_0^1 A_i di = 1$.

We also assume log utility over consumption and a Cobb-Douglas production function because it simplifies the exposition and clarifies the key mechanisms at play. However, all the proofs in Appendix A cover the more general case of a constant relative risk aversion (CRRA) utility and CES production function. These proofs reveal that most of the results from the basic model do not change in the more general CRRA-CES model.

B. Shocks

In the first period, $t = 0$, the economy is subject to several shocks. In the final good production function, there are demand shocks for the intermediate good:

[7] As is standard, the money in the utility function term is readily interpreted as the effort spent shopping. Namely, it is equivalent to (i) rescaling the utility over the final good by $1 + \chi$ for some $\chi > 0$ and (ii) assuming that, in order to shop for C_t unit of final goods with real balance M_{t-1}/P_t, a shopper suffers a disutility $-1/V \log(M_{t-1}/P_t) + \chi \log(C_t)$. Separable utility between money and consumption also implies that money has no real effects in the economy under perfect information; i.e., money is neutral. As we will show below, under dispersed information, that is not the case anymore, and monetary shocks will be expansionary.

[8] Note that (1) is the unitary-elasticity limit of a constant elasticity of substitution (CES) production function, where A_i is a multiplicative shock to intermediate good i in the aggregator.

$$\log A_i = a_i - \frac{1}{2\psi_a},$$

where a_i is independent and identically distributed (iid) across sectors and normally distributed with mean zero and variance $1/\psi_a$. These shocks translate into idiosyncratic shocks to the demand for intermediate goods.[9]

There are also productivity shocks that affect the intermediate production,[10]

$$\log \Theta_i \equiv \theta_i = \theta + \varepsilon_i,$$

where θ is the *aggregate* component of the productivity shocks, which is assumed to be normally distributed with mean normalized to zero and variance $1/\Psi_\theta$, and ε_i is the *idiosyncratic* component of the productivity shock, which is assumed to be normally distributed with mean zero and variance $1/\psi_\theta$.

On the monetary side, we let the aggregate money supply be constant and equal to M, and we assume that there is a random "velocity" disturbance, affecting the utility of real money holdings:[11]

$$\log V = \mu_v + v,$$

where v represents a money velocity shock, normally distributed with mean zero and variance $1/\Psi_v$. For simplicity, we assume that all shocks are permanent.

All of these shocks play a specific role in the analysis that follows. The velocity shock, v, generates uncertainty about the aggregate nominal expenditures. The aggregate component of the productivity shocks affecting the intermediate good sectors, θ, generates uncertainty regarding the average output in the economy and makes the aggregate price level only partially revealing of the velocity shock. Finally, the idiosyncratic demand shocks confuse intermediate goods workers as to the actual source of the relative price changes they observe in their sector.

We will measure the amount of information in precision units: public releases of exogenous information about v and θ translate into increases in Ψ_v and Ψ_θ. In order to study the effect of public information releases,

[9] Note that we do not impose aggregate shocks in the final good sector. However, this is without loss of generality because in a linear equilibrium, such shocks will be revealed to all agents through the difference between the average intermediate price and the final good price. As a result, adding such a shock would not change our welfare analysis.

[10] Burstein and Hellwig (2007) argue that both demand shocks and productivity shocks are necessary to match micro evidence on comovements between intermediate goods prices and quantity.

[11] These shocks can be interpreted as shocks to the shopping technology discussed in n. 7. The assumption that the aggregate money supply is constant is not particularly important; i.e., we could have assumed instead that the monetary aggregate will follow a particular deterministic process.

we therefore conduct comparative static exercises with respect to the exogenous parameters Ψ_v and Ψ_θ.[12]

C. *Timing and Information Structure*

The timing of events and decisions is as follows. At the beginning of every period, the family separates into a shopper, a final good producer, and a continuum of workers. While these different family members can observe nominal prices, they cannot communicate with each other until the end of the period. Specifically, we assume that family members observe all nominal prices in the economy before making their decisions, but they observe only the realization of the shocks that are directly relevant to their own decisions. That is,

- the shopper observes the velocity shock, v, of this period before deciding how much of the final good to buy and how much money to carry over to the next period;
- the producer of the final goods observes the demand shocks, a_i, before deciding how much of the final good to produce; and
- each worker i, who produces intermediate good i, observes the sectoral productivity shock θ_i before deciding how much labor to supply.

At the end of the period, all family members come together and share their information. Given that shocks are permanent, this implies that there remains no uncertainty regarding any of the shocks affecting the economy from period $t \geq 1$ onward.

One feature of the model that is worth emphasizing is that all the shocks affecting the economy are fundamental in nature. In particular, there is no exogenous noise blurring the observation of prices.

III. Definition of an Equilibrium

An equilibrium is made up of a sequence of final goods prices and production levels, distributions of labor supplies and production across sectors, and distributions of intermediate prices in the economy, such that at each point in time (1) family members maximize the family's utility, given their information about shocks (described above) and the

[12] One should bear in mind that this exercise changes the *conditional variance* of the shock. The *unconditional variance*, which is the fundamental volatility of the shocks, should be kept constant. In the language of our model, this amounts to changing the precision of the posterior while keeping the precision of the prior the same. To simplify the exposition, we do not make this distinction explicit in the main text of the paper, but all proofs carefully distinguish between the precisions of the common prior and the ones of any additional public signal.

observation of all nominal prices in the economy; and (2) the final goods market clears, $Y_t = C_t$, all intermediate goods markets clear, $Y_{it} = \Theta_i L_{it}$ for all $i \in [0, 1]$, and the money market clears, $M = M_t^d$.

Before we formally characterize an equilibrium, it is convenient to first analyze the family's problem and to show that we can simply concentrate on the first-period problem because the economy has a simple unique equilibrium from period $t \geq 1$ onward.

A. *Solving the Family's Problem*

Now we proceed to solve the problem of each family member and to show that we can concentrate our analysis to the initial period, $t = 0$.

1. The Shopper's Decision

After the end of the first period, $t = 0$, all shocks are known by all agents in the economy. Thus, for all $t \geq 1$, if we let λ_t denote the Lagrange multiplier on the household's sequential budget constraint, then the first-order condition with respect to M_t^d delivers

$$\frac{\lambda_t}{P_t} = \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] + \beta \frac{1}{MV},$$

where we have used the money market–clearing condition $M_t^d = M$. In Appendix A (Sec. A), we apply Obstfeld and Rogoff's (1983) argument for ruling out implosive or explosive solutions, and we show that

$$\frac{\lambda_t}{P_t} = \frac{\beta}{1-\beta}\frac{1}{MV}$$

is the only solution consistent with an equilibrium for $t \geq 1$.

In the first period, $t = 0$, the shopper chooses consumption without knowing the exact decisions made by the workers in each sector and the final good producer: in principle, the shopper is uncertain about how much real resources he can spend, on the right-hand side of the family budget constraint. However, in the equilibrium that we solve for, all aggregate variables are functions of the two aggregate shocks, v and θ. In particular, the log of the final good price, observed by the shopper, is an affine function of (v, θ): given that the shopper knows v, he will be able to infer θ perfectly.[13] Hence, the shopper knows about all aggregate shocks hitting the economy and effectively makes a decision under full information. In anticipation of this, the shopper's first-order condition with respect to money balances at time $t = 0$ becomes

[13] Section H of App. A rules out the possibility that the final good price is an affine function of v only.

$$\frac{\lambda_0}{P_0} = \beta\frac{\lambda_1}{P_1} + \beta\frac{1}{MV} = \frac{\beta}{1-\beta}\frac{1}{MV}.$$

The shopper's first-order condition with respect to consumption is then

$$\lambda_t = \frac{1}{C_t},$$

and after substituting in $C_t = Y_t$, we find that the quantity equation

$$P_t Y_t = \frac{1-\beta}{\beta} MV \tag{2}$$

holds for all $t \geq 0$.

2. The Final Good Producer's Decision

The final good producer maximizes the value of his profits to the household. After observing the prices in the economy, together with the distribution of demand shocks, $\{a_i\}_{i \in [0,1]}$, the final good producer maximizes

$$\Pi_t = P_t Y_t - \int_0^1 \frac{P_{it}}{P_t} Y_{it} di$$

subject to the Cobb-Douglas production technology (1), which delivers a demand function for intermediate goods:

$$P_t A_i \frac{Y_t}{Y_{it}} = P_{it}. \tag{3}$$

As expected, given constant returns to scale, equilibrium prices will guarantee that the profits of the final good sector are zero.

3. The Worker's Decision

The problem of an intermediate good worker is to maximize her contribution to the family's utility, which can be written as

$$\mathbb{E}_{it}\left[\lambda_t \frac{P_{it}}{P_t}\Theta_i L_{it} - \frac{\delta}{1+\delta} L_{it}^{1+1/\delta}\right],$$

where the expectation is with respect to the equilibrium information set of the worker (to be derived later), and where λ_t is the marginal utility of consumption, which might not be known by the worker with certainty at the moment of the labor supply decision.

Now, from

$$\frac{\lambda_t}{P_t} = \frac{\beta}{1-\beta}\frac{1}{MV}$$

and $Y_{it} = \Theta_i L_{it}$, the first-order condition of the worker yields

$$Y_{it} = \Theta_i \left[\frac{\beta}{1-\beta} \mathbb{E}_{it} \left[\frac{1}{MV} \right] \Theta_i P_{it} \right]^{\delta}, \tag{4}$$

where we have taken out of the expectation the two variables that the worker can observe: the price, P_{it}, and the sectoral shock, Θ_i.

Even though the expectation of a velocity shock appears in the labor supply equation, it is possible to rewrite the equation to show that what is driving the worker's labor supply decision is the expectation of the real sectoral demand shock, A_i. To see this, note that after plugging in $\lambda_t = 1/Y_t$ and using the sectoral demand (3), we could rewrite the worker's labor supply as

$$L_{it} = \mathbb{E}_{it}[A_i]^{\delta/(1+\delta)}. \tag{5}$$

The equivalence between (4) and (5) sheds light on the signal extraction problem driving a worker's labor supply decision. For given sectoral productivity, Θ_i, and expected velocity shock, $\mathbb{E}_{it}[1/V]$, the worker infers that an increase in local price, P_{it}, must have been generated by an increase in her sectoral demand shock, A_i, prompting her to increase her labor supply. Similarly, for a given $\mathbb{E}_{it}[1/V]$, a supply shock induced by higher sectoral productivity, Θ_i, with no decrease in local price, P_{it}, leads the worker to infer that her sectoral demand shock A_i is higher, which increases her labor supply. Finally, for a given sectoral productivity, Θ_i, a higher expected velocity shock, $\mathbb{E}_{it}[1/V]^{-1}$, with no increase in local price, P_{it}, leads the worker to infer that the sectoral demand shock A_i is lower, which decreases her labor supply.

Also, equation (5) reveals a distinguishing feature of our setup with log utility and Cobb-Douglas final good production: a worker's labor supply is only a function of her beliefs about her sectoral shock and is not otherwise affected directly by what other agents in the economy are doing. More specifically, one can show that payoffs are separable: by using the aggregate production function together with $Y_{it} = \Theta_i L_{it}$, we can write the welfare of the family, ignoring the utility from real money balances, as

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \left[A_i \log(\Theta_i L_{it}) - \frac{\delta}{1+\delta} L_{it}^{1+1/\delta} \right] di \right\} \right]. \tag{6}$$

Expression (6) shows that the impact of a worker's labor supply decision on welfare does not directly depend on what others do. Note as well that equation (5) corresponds to the labor supply decision that maximizes (6) if one were to assume that workers' information sets are exogenously given.[14] From the work of Angeletos and Pavan (2007), it follows that public information would always be beneficial in our framework *if the information structure were exogenous.* As we will show below, this result will be overturned once agents are allowed to learn from each other through prices, making the information structure endogenous.

B. *Linear Equilibrium in the First Period*

Without loss of generality, let us normalize the parameters so that $e^{\mu_v}M(1-\beta)/\beta = 1$. Note that for $t \geq 1$, there is no uncertainty in the economy, and there exists a unique solution to equations (1), (2), (3), and (4) that characterizes the unique equilibrium from time $t \geq 1$. We then focus our analysis on the time $t = 0$, and we drop the time subscripts to economize on notation.

Assuming that at $t = 0$ the cross-sectional distribution of log intermediate output is normal and the posterior beliefs of the worker about the velocity shock are normally distributed with mean $\mathbb{E}_i[v]$ and variance $V_i[v]$, we can write equations (1), (2), (3), and (4) in log-linear form:

$$\text{quantity equation:} \quad y = v - p, \tag{7}$$

$$\text{intermediate goods demand:} \quad y_i = y + a_i - \frac{1}{2\psi_a} + p - p_i, \tag{8}$$

$$\text{aggregate output:} \quad y = \int_0^1 y_i di + \int_0^1 y_i a_i di, \tag{9}$$

$$\text{intermediate goods supply:} \tag{10}$$

$$y_i = (1+\delta)\theta_i + \delta\left(p_i - \mathbb{E}_i[v] + \frac{V_i[v]}{2}\right),$$

where lowercase variables indicate natural logs.[15]

[14] To see this, note that eq. (6) can be written as

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\int_0^1 \mathbb{E}_{it}\left[A_i \log(\Theta_i L_{it}) - \frac{\delta}{1+\delta}L_{it}^{1+1/\delta}\right]di\right\}\right],$$

and eq. (5) follows from differentiating with respect to L_{it}.

[15] For the final good production function, we use the fact that

$$\log Y = \int_0^1 A_i \log Y_i di = \int_0^1 y_i di + \int_0^1 (e^{a_i - 1/2\psi_a} - 1) y_i di$$

$$= \int_0^1 y_i di + \operatorname{Cov}(e^{a_i - 1/2\psi_a}, y_i).$$

Then when we use that $\operatorname{Cov}(e^{z_i}, x_i) = \mathbb{E}[e^{z_i}]\operatorname{Cov}(z_i, x_i)$ if both x_i and z_i are normal, the result follows.

The local supply function (10) is similar to that of Lucas (1973), with the same implication that monetary shocks are expansionary (see online App. B). However, there is one key difference. Motivated by his earlier work, Lucas starts from a local supply that increases with the difference between the local price and the local expectation of the *aggregate price level* in the economy. In our model, in contrast, there is no uncertainty about the aggregate price level: all workers observe it perfectly. Still, uncertainty about the nominal shock matters: namely, the local supply increases with the difference between the local price and the local expectation of the *aggregate velocity shock.* Thus, in our model, the aggregate supply will be an increasing function of the difference between the price level and the average of the sectoral expectations of the aggregate velocity shock.

Borrowing from the literature on noisy rational expectations in financial markets (see, among many others, Grossman [1975] and Hellwig [1980]), we restrict ourselves to symmetric linear equilibria, as in the following definition.[16]

DEFINITION 1. A symmetric linear equilibrium is a final good price, p, aggregate output, y, and a distribution of intermediate goods prices, $\{p_j\}_{j\in[0,1]}$ and intermediate good supplies $\{y_j\}_{j\in[0,1]}$, such that the following holds:

i. Log prices are linear functions of the states

$$p = K_0 + K_v v + K_\theta \theta \quad \text{and} \quad p_j = k_0 + k_v v + k_\theta \theta + \eta_j, \qquad (11)$$

for some constant $(K_0, K_v, K_\theta, k_0, k_v, k_\theta)$, where either $K_\theta \neq 0$ or $k_\theta \neq 0$ and η_j is a mean zero linear combination of sector-specific shocks, a_j and ε_j.

ii. Workers' expectations are rational: after the private signal θ_i and all nominal prices are observed, the posterior belief of the worker is normally distributed with mean

$$\mathbb{E}_i[v] = \mathbb{E}[v|p, \{p_j\}_{j\in[0,1]}, \theta_i]$$

and variance $V_i[v] = \text{Var}\,[v|p, \{p_j\}_{j\in[0,1]}, \theta_i]$.

iii. Agents' decisions are optimal and markets clear; that is, equations (7), (8), (9), and (10) are satisfied.

The restriction that $K_\theta \neq 0$ or $k_\theta \neq 0$ ensures that the shopper can infer the exact realization of θ when observing nominal prices, a guess we made in Section III.A. Although we stated this requirement as an

[16] There are not many results in the literature that deal with the existence of nonlinear equilibria in economies with asymmetric information. The most closely related work is DeMarzo and Skiadas (1998), which characterizes the uniqueness of the fully revealing equilibrium in quasi-complete economies. These results, however, cannot be applied to our environment.

equilibrium condition for simplicity, Section H of Appendix A shows that this guess is without loss of generality: it must hold in any symmetric linear equilibrium.

We conclude this section by showing that, in a symmetric linear equilibrium, nominal prices partially reveal the aggregate shocks, (v, θ). Indeed, after averaging the intermediate goods demand equation (8) across all sectors $j \in [0, 1]$ and plugging in the aggregate output equation (9), one finds

$$p = \int_0^1 p_j dj + \frac{1}{2\psi_a} - \int_0^1 y_j a_j dj. \tag{12}$$

This means that the final good price is equal to the average of intermediate goods prices, up to the second moments, $1/\psi_a$ and $\int_0^1 y_j a_j dj$, which do not depend on the realizations of θ and v. This immediately implies the following lemma.[17]

LEMMA 1 (Nominal prices are partially revealing). In a symmetric linear equilibrium, $k_v = K_v$ and $K_\theta = k_\theta$.

Specifically, the observation of nominal prices alone does not reveal the exact realizations of v and θ to workers, but only the linear combination $K_v v + K_\theta \theta$.

IV. Equilibrium Characterization

The standard approach to finding linear equilibria is to use the method of undetermined coefficients. One starts by computing the workers' expectations of the velocity shock, conditional on the belief that prices are given by (11). Next, given these expectations, one solves for prices using the system of equilibrium equations (7)–(10). Consistent with (11), these prices turn out to be affine functions of the shocks, but with a new vector of coefficients that is a function of the vector of coefficients posited in (11). Of course, in an equilibrium, the two vectors of coefficients have to be the same. Solving the resulting fixed-point equation yields the following proposition.

PROPOSITION 1. Let $\Omega \equiv -K_v/K_\theta$. The set of symmetric linear equilibria is nonempty, and it is composed of elements $(p, \{p_i\}_{i\in[0,1]}, y, \{y_i\}_{i\in[0,1]})$ such that, for every Ω solving the fixed-point equation,

$$\Omega = \frac{1}{1+\delta} + \frac{\delta}{1+\delta}\frac{\psi_a + \Omega^2\psi_\theta}{\Psi_v + \Omega^2\Psi_\theta + \psi_a + \Omega^2\psi_\theta}, \tag{13}$$

log prices p and $\{p_i\}_{i\in[0,1]}$ are given according to (11) with coefficients

[17] More generally, in App. A, we show that this property holds for any CES constant returns to scale production function.

$$K_0 = \frac{\delta}{2(1+\delta)}\left(\frac{1}{\Psi} - \frac{1}{\psi_a}\right) \quad \text{and} \quad k_0 = -\frac{1}{2(1+\delta)}\left(\frac{1}{\psi_a} + \frac{\delta}{\Psi}\right),$$

$$K_\theta = k_\theta = -\frac{K_v}{\Omega} \quad \text{and} \quad K_v = k_v = 1 - \frac{\delta}{1+\delta}\frac{\Psi_v}{\Psi},$$

$$\eta_i = \frac{1}{1+\delta}\left(1 + \delta\frac{\psi_a}{\Psi}\right)\left(a_i + \frac{\varepsilon_i}{\Omega}\right),$$

where $\Psi \equiv \Psi_v + \Omega^2\Psi_\theta + \psi_a + \Omega^2\psi_\theta$. Log quantities y and $\{y_i\}_{i\in[0,1]}$ are the unique solutions to (7) and (8) given log prices.

That is, for any solution to equation (13), there exists a unique linear equilibrium such that $-K_v/K_\theta = \Omega$, and vice versa. Note too that equation (13) has at least one solution and that all solutions of (13) lie within the interval $[1/(1+\delta), 1]$.

The ratio $\Omega = -K_v/K_\theta$ is the central endogenous variable in our analysis of informational externalities. To see why, note that, after we subtract K_0 and divide by $K_\theta \neq 0$, it follows that observing the final good price is equivalent to observing the signal $v - \theta/\Omega$. Thus, the value of Ω ultimately determines the endogenous informativeness of the final good price about velocity: when it is large, the price is very informative about velocity and vice versa when it is small.

Explaining the Fixed-Point Equation. In the rest of this section, we propose a heuristic derivation of the fixed-point equation (13). Although this heuristic derivation is admittedly more roundabout than the one outlined above, it has the advantage of building intuition about the two-way interaction between workers' labor supply decision and the information aggregated by prices.

The information set.—The first step is to transform the worker's information set into a collection of conditionally independent public and private signals about v. We first note that we can eliminate the distribution of other sectors' intermediate goods prices from the information set because it provides redundant information about v. Indeed, for the purpose of forecasting v, the average intermediate good price, $\int_0^1 p_j dj$, is a sufficient statistic for the entire distribution, $\{p_j\}_{j\neq i}$. But, by lemma 1, the average intermediate good price yields the same information as the final good price, p, which is already in the worker's information set.

Next, we observe that, from the intermediate good price p_i in the worker's own sector, the worker infers a signal equivalent to the nominal demand, $v + a_i$. To see this, we equate the intermediate goods supply and demand, (10) with (8), use the quantity equation (7) and obtain

$$p_i = \frac{1}{1+\delta}\left(v + a_i - \frac{1}{2\psi_a}\right) + \frac{\delta}{1+\delta}\left(\mathbb{E}_i[v] - \frac{V_i[v]}{2}\right) - \theta_i. \qquad (14)$$

Because worker i observes θ_i and "knows his own expectations," $\mathbb{E}_i[v]$, it follows intuitively that observing the intermediate good price, p_i, is equivalent to observing the nominal demand, $v + a_i$.

Finally, note that because a worker already observes the final good price, which is observationally equivalent to $v - \theta/\Omega$, she can replace the signal $\theta_i = \theta + \varepsilon_i$ by the signal $\theta_i/\Omega + v - \theta/\Omega = v + \varepsilon_i/\Omega$. That is, because the worker has private information about the technological "noise" term in the final good price, θ, she can back out an endogenous private signal about velocity, $v + \varepsilon_i/\Omega$.

In summary, for the purpose of forecasting velocity, the worker's information set is equivalent to the three conditionally independent signals,$v + a_i$, $v - \theta/\Omega$, and $v + \varepsilon_i/\Omega$. In particular, observing the final good price affects a worker's information in two ways. It endogenously increases her public information, through the signal $v - \theta/\Omega$. But it also endogenously increases her private information, through the signal $v + \varepsilon_i/\Omega$.

The worker's forecast of velocity.—With conditionally independent and normally distributed signals, Bayes' rule is straightforward: the worker's forecast of v is just a convex combination of the signals and the prior mean, with convex weights reflecting the relative precision of each.

In anticipation of our discussion of information aggregation, it is convenient to group terms in the worker's forecast as follows. We let the *private forecast* of v be the expectation of v conditional on the two private signals, $v + a_i$ and $v + \varepsilon_i/\Omega$, given a fully diffuse prior; and we let the *public forecast* be the expectation of v conditional on the public signal $v - \theta/\Omega$, given the common prior. Based on this grouping, the forecast of v is then

$$\mathbb{E}_i[v] = \omega \times \text{private forecast} + (1 - \omega) \times \text{public forecast}, \tag{15}$$

where

$$\omega = \frac{\psi_a + \Omega^2\psi_\theta}{\Psi_v + \Omega^2\Psi_\theta + \psi_a + \Omega^2\psi_\theta}. \tag{16}$$

The convex weight ω is, as usual, increasing in the precision $\psi_a + \Omega^2\psi_\theta$ of the worker's private forecast, that is, in the precision of her overall private information. Symmetrically, the weight ω is decreasing in the precision $\Psi_v + \Omega^2\Psi_\theta$ of the public forecast.

How price aggregates private information.—After substituting the average intermediate good price of equation (14) into equation (12) and ignoring second moments that are constant, we find that

$$p = \frac{1}{1+\delta}v + \frac{\delta}{1+\delta}[\omega \times \text{average private forecast} \\ + (1-\omega) \times \text{public forecast}] - \theta,$$

where average private forecast is equal to v. This is where separating the expectations into a public and a private forecast becomes useful. First, the cross-sectional average private forecast is just equal to v because it is based on iid signals and a fully diffuse prior. Second, because the public forecast is known to everyone, a worker can subtract it from the final good price. Taken together, this implies that observing the final good price is indeed equivalent to observing a signal of the form $v - \theta/\Omega$, where

$$\Omega = \frac{1}{1+\delta} + \frac{\delta}{1+\delta}\omega. \tag{17}$$

Equation (16) showed how workers' weighting decision, ω, depends on the endogenous informativeness of the price level, Ω. Equation (17) closes the loop: it shows how the endogenous informativeness of the price level, Ω, increases in workers' weight on private information, ω. Combining the two equations, we obtain the fixed-point equation of the proposition.

V. Public Information and Welfare

In this section, we analyze the welfare effect of public information releases about v and θ. Public information has a direct beneficial effect: with the endogenous informativeness of prices, Ω, taken as given, it increases the total knowledge of workers and allows for more informed decisions. However, it also has the adverse effect of reducing the weight that workers put on their private information, thus reducing the endogenous informational content of nominal prices. We show below that the second effect can dominate.

A. *Welfare Criterion and Equilibrium Selection*

We take our criterion to be utilitarian welfare: the ex ante utility of the representative family. In our model with log utility and a Cobb-Douglas production function, welfare can be shown to be an increasing function of the workers' posterior precision about v. That is, the family is better off when its workers know more.

Proposition 2. In a symmetric linear equilibrium, public information increases ex ante utilitarian welfare if and only if it increases the posterior precision about v, $\psi_a + \Omega^2\psi_\theta + \Psi_v + \Omega^2\Psi_\theta$.

Although intuitive, the previous result is not a forgone conclusion: as is well known, information does not necessarily have a positive social value.[18]

[18] Perhaps the best-known example is from Hirshleifer (1971), which shows that information destroys insurance opportunities. See also Brunermeier (2001, chap. 1) and the references therein.

While proposition 1 establishes the existence of a linear equilibrium, our model can admit multiple linear equilibria (for the full characterization, see online App. B). This possibility introduces a standard difficulty for welfare analysis. Indeed, one has to decide on which equilibrium households will coordinate, and different equilibria often admit opposite comparative statics. In what follows, we focus on the highest welfare equilibrium so that we abstract from the negative welfare impact of coordination failure. Moreover, our main welfare result does not depend on multiplicity: we show in online Appendix B that it holds in regions of the parameter space where the equilibrium is unique. Proposition 2 immediately implies the following lemma.

LEMMA 2. The highest welfare equilibrium corresponds to the largest solution, Ω_*, of the fixed-point equation (13).

B. *U-Shaped Welfare and Bang-Bang Communication*

Recall first that, in equilibrium, a worker's posterior precision about v is

$$\psi_a + \Omega_*^2\psi_\theta + \Psi_v + \Omega_*^2\Psi_\theta. \tag{18}$$

With Ω_* held constant, both Ψ_v and Ψ_θ increase public knowledge. This is the intuitive direct beneficial effect of public information: it directly increases knowledge about v or it increases knowledge about θ, which allows households to extract more information about v from nominal prices.

However, there is a countervailing equilibrium effect: after an increase in public information about either v or θ, households put less weight on their private forecast, reducing Ω_*. This indirect effect tends to decrease the informational content of prices and to reduce workers' posterior precision about v.

LEMMA 3. In the highest welfare equilibrium, Ω_* (i) is strictly decreasing in Ψ_v and Ψ_θ, (ii) goes to $1/(1 + \delta)$ as either Ψ_v or Ψ_θ goes to infinity, and (iii) goes to one if both Ψ_v and Ψ_θ go to zero.[19]

From the fixed-point equation, (13), it is possible to obtain an equation for workers' posterior precision:

$$\psi_a + \Omega_*^2\psi_\theta + \Psi_v + \Omega_*^2\Psi_\theta = \frac{\delta}{1+\delta}\,\frac{\psi_a + \Omega_*^2\psi_v}{\Omega_* - [1/(1+\delta)]}, \tag{19}$$

where the left-hand side is the total posterior precision of the worker's beliefs. Note that a change in Ψ_θ or Ψ_v affects the right-hand side only

[19] Although lemma 3 does not discuss it, increases in Ψ_v or Ψ_θ could cause negative discontinuities in Ω_* because of the multiplicity of equilibria and our equilibrium selection. Such negative discontinuities in Ω_* create, obviously, discrete welfare losses. See online App. B for a detailed analysis.

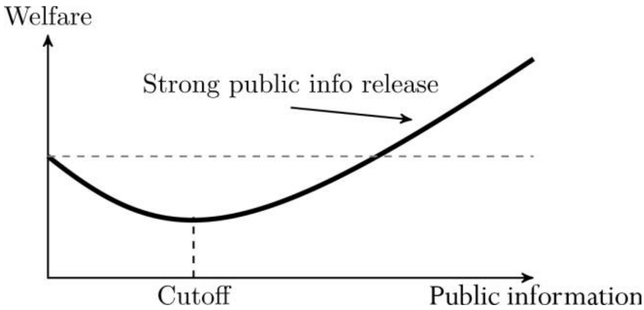

FIG. 1.—U-shaped welfare. When public information is low, Ω_* is above Ω_{cutoff}, and welfare decreases in public information. Conversely, when public information is high, Ω_* is below Ω_{cutoff}, and welfare increases in public information.

through the effect on the equilibrium Ω_*. Hence, we can evaluate the welfare effects of an increase in Ψ_θ and/or Ψ_v by studying the right-hand side as a function of Ω_*. In particular, after taking derivatives, one sees that the right-hand side is a U-shaped function of Ω_*: it increases with Ω_* if

$$\Omega_* > \Omega_{\text{cutoff}} = \frac{1}{1+\delta} + \sqrt{\frac{\psi_a}{\psi_\theta} + \left(\frac{1}{1+\delta}\right)^2} \qquad (20)$$

and it decreases for the opposite strict inequality. But, by lemma 3, Ω_* is itself a strictly decreasing function of (Ψ_v, Ψ_θ). Since the composition of a U-shaped function with a decreasing function is also a U-shaped function, it follows that welfare is also a U-shaped function of (Ψ_v, Ψ_θ). In particular, welfare may decrease when (Ψ_v, Ψ_θ) is small if the resulting Ω_* is above the cutoff, Ω_{cutoff}, of equation (20). Figure 1 illustrates and immediately implies our next result.

PROPOSITION 3 (Bang-bang communication). Suppose that the government has several independent signals about v and θ that would increase public precisions if revealed. Then the optimal communication policy is to announce all or none.

In this setup, this means that either full transparency or full opacity is optimal: selectively picking which information to announce, or revealing only part of the available information, always will be suboptimal. Our next proposition studies some conditions for transparency and opacity to be optimal.

PROPOSITION 4. A sufficiently large release of public information about v of θ will always increase welfare. Consider, however, any given finite increase in public information. Then there exists (ψ_a, ψ_θ) such that this increase is welfare decreasing if and only if $\delta > 1$.

The first point of the proposition intuitively arises from the fact that, when Ψ_v goes to infinity, then the posterior precision about v goes to infinity as well, and welfare is maximized. The same is true if Ψ_θ goes to infinity, as observing the final good price yields arbitrarily precise information about v. To intuitively derive the second point of the proposition, note that, if $\delta \leq 1$, then $\Omega_{\text{cutoff}} > 1$. But since $\Omega_* < 1$, this implies that Ω_* can never be above Ω_{cutoff} and thus that public information always increases welfare. Next, if $\delta > 1$, then Ω_{cutoff} is strictly less than one if the ratio ψ_a/ψ_θ is small enough. At the same time, given any Ψ_v or Ψ_θ, then if the level of ψ_a and ψ_θ is large enough, it follows from the fixed-point equation (13) that Ω_* can be made arbitrarily close to one. Therefore, any release of public information up to Ψ_v or Ψ_θ will reduce welfare.

VI. Explaining the Negative Welfare Result

Our negative welfare results rely on two features of the model: the elasticity of workers' labor supply and the fact that workers acquire some private information from prices. Below we discuss why this is the case.

A. *The Role of Labor Supply Elasticity*

Proposition 4 shows that for public information to be welfare reducing, workers' labor supply elasticity has to be large enough. As we show below, the reason is that this elasticity governs whether the informational content of prices is predominantly determined by the demand of the fully informed final good firms or by the supply of imperfectly informed intermediate good workers.

Recall that the fixed-point equation was obtained by showing that the final good price could be written as

$$p = \frac{1}{1+\delta}v + \frac{\delta}{1+\delta}\int_0^1 \mathbb{E}_i[v]di - \theta + \text{constants}.$$

That is, the Frisch elasticity parameter controls the weight of the price level on exogenous information about v (the v term in the equation above) versus endogenous information about v (the $\int \mathbb{E}_i[v]di$ term).

The reason for this lies in the way the market for intermediate goods clears. Consider first the case of a low labor supply elasticity (i.e., δ close to zero), illustrated in figure 2*a*. Then the supply of intermediate good i is inelastic, approximately equal to θ_i, and the price p_i adjusts so that final good firms are willing to absorb it:

$$p_i \simeq v + a_i - \frac{1}{2\psi_a} - \theta_i.$$

The intermediate good price still reveals information about v because the nominal demand of the final good firms depends on economywide

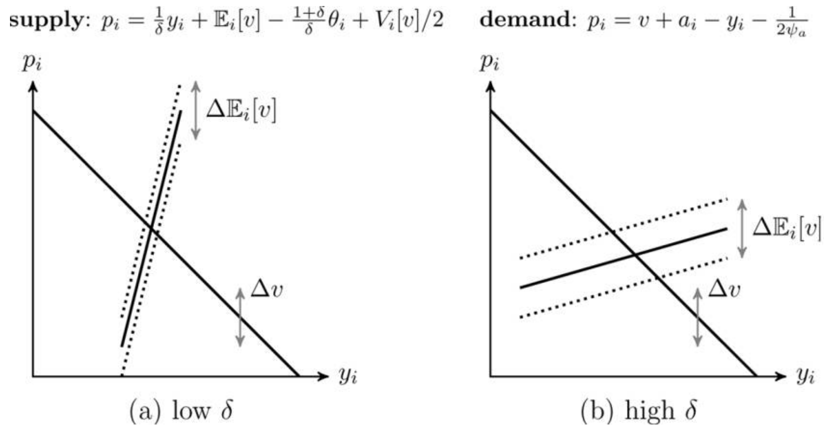

FIG. 2.—Information aggregation and the intermediate good market. When the Frisch elasticity of labor supply is low (panel *a*), the price that clears the intermediate good market will be determined mainly by the demand curve, and thus variations in p_i mainly reflect variations in v. When the Frisch elasticity of labor supply is high (panel *b*), the price will be determined mainly by the supply curve, and thus the beliefs of workers, and thus variations in p_i, mainly reflect variations in $\mathbb{E}_i[v]$.

nominal income, $p + y = v$. But because workers' expectations do not feed back into the price, the information content of the intermediate good price is essentially exogenous. In this situation, public information has no significant adverse effect on the informativeness of nominal prices and on welfare.

Next, consider the opposite case of high labor supply elasticity, illustrated in figure 2*b*. Recall that, in equilibrium, all intermediate goods are produced in positive and finite quantity, implying that the solution to workers' optimal labor supply problem has to be interior. But a high labor supply elasticity means that the worker's disutility of labor is close to linear, so to ensure an interior solution, the marginal cost of labor has to be approximately equal to the workers' expected marginal contribution to the family's income. This leads to the intermediate good price

$$p_i \simeq \mathbb{E}_i[v] - \frac{V_i[v]}{2} - \theta_i,$$

which is unit elastic with respect to the worker's expectation of v. Thus, workers' expectations feed back into nominal prices with a high elasticity, making the informational content of the price essentially endogenous. Public information now can have negative effects on this informativeness, the basis for the negative welfare result.

For negative welfare effects to arise, the elasticity parameter, δ, has to be greater than one. There is a long-standing debate in quantitative macroeconomics regarding the value of this elasticity parameter. If δ is interpreted as the micro elasticity of labor supply, then the data suggest that it should be smaller than one. But if it is interpreted as the macro elasticity, the data suggest that it should be much larger than one. In order to understand better which elasticity matters for welfare, online Appendix B presents a variant of our model with frictional unemployment, along the lines of Hall (2009) and Shimer (2010), and in which the micro elasticity parameter differs from the macro elasticity. In a counterpart to proposition 4, we show that the elasticity that matters for welfare is the empirically larger macro elasticity. This is intuitive given the above discussion since the macro elasticity governs the response of total hours to private information and thus determines the strength of the learning externality. If one were to use the micro elasticity instead of the macro elasticity in the analysis, then one would obtain the wrong equilibrium equations and the wrong welfare prescriptions.

B. *The Role of Private Learning from Price*

When workers learn private information from prices, their actions can become strategic complements. This complementarity amplifies the negative effect of public information and is key for the negative welfare result.

Recall that in our model with log utility and a Cobb-Douglas production function, there are no direct complementarities or substitutabilities in actions: from equation (5), a worker's labor supply can be viewed as a function of her beliefs about her sectoral shock, A_i, and is not otherwise directly affected by what other agents in the economy are doing. We now show that complementarities or substitutability nevertheless may arise indirectly because workers are learning from prices, and the informativeness of prices is affected by what others are doing.

To make this point it is helpful to recall that a worker's individually optimal weight on private information is given by

$$\omega = H(\Psi_v + \Omega^2\Psi_\theta, \psi_a + \Omega^2\psi_\theta), \quad (21)$$

where, from the best reply function (21), $H(X, x) \equiv x/(x + X)$, $X = \Psi_v + \Omega^2\Psi_\theta$ is the precision of a worker's public forecast, and $x = \psi_a + \Omega^2\psi_\theta$ is the precision of her private forecast.

Now suppose that all workers were forced to increase the weight ω on their private forecasts. Equation (17) implies that the endogenous informativeness of the price level, Ω, increases. This, in turn, increases the precisions X and x of both the public and the private forecasts. How would an individual worker react to such an aggregate change? There are two opposite effects on a worker's individually optimal weight. The first effect, which follows because $\partial H/\partial X < 0$, tends to decrease ω. Indeed, when Ω increases, the precision of the public forecast, $X = \Psi_v + \Omega^2\Psi_\theta$, increases. With the precision of the private forecast, $x = \psi_a + \omega^2\psi_\theta$, held constant, an individual worker would find it optimal to rely more on the improved public forecast and less on her private one, that is, to decrease ω. This is a force for *strategic substitutability.* However, because $\partial H/\partial x > 0$, there is also an opposite effect. Indeed, when Ω increases, the precision of the private forecast, x, increases. With the precision of the public forecast, X, held constant, an individual worker would find it optimal to rely more on her improved private forecast, that is, to increase ω. This is a force for *strategic complementarity.*

The complementarities play a key role in the welfare analysis because they are necessary for public information to be welfare reducing. To demonstrate this, we start with a hypothetical setup that suppresses complementarities by assuming that observation of the price level does not affect the precision of the private forecast.

LEMMA 4. With $x \equiv \psi_a + \Omega_*^2\psi_\theta$ held the same, (1) the equilibrium weight $\Omega_\diamond$, which solves

$$\Omega_\diamond = \frac{1}{1+\delta} + \frac{\delta}{1+\delta} H(\Psi_v + \Omega_\diamond^2\Psi_\theta, x),$$

is a decreasing function of Ψ_θ and Ψ_v: $\partial\Omega_\diamond/\partial\Psi_\theta < 0$ and $\partial\Omega_\diamond/\partial\Psi_v < 0$; and (2) the posterior precision $x + \Psi_v + \Omega_\diamond^2\Psi_\theta$ is an increasing function of Ψ_θ and Ψ_v.

The lemma shows that when $x = \psi_a + \Omega_*^2\psi_\theta$ is held constant, even though public information about θ or v reduces $\Omega_\diamond$, it always increases workers' posterior precision about v: $x + \Psi_v + \Omega_\diamond^2\Psi_\theta$. The intuition for this result lies in the fact that the precision of the public forecast $\Psi_v + \Omega_\diamond^2\Psi_\theta$ cannot decrease with an increase in (Ψ_v, Ψ_θ) given that x has remained constant. Indeed, if the precision of the public forecast were to decrease, then because $\partial H/\partial X < 0$, workers would find it optimal to rely more on their private information, and the equilibrium $\Omega_\diamond$ would have to increase. This in turn implies an increase in $\Psi_v + \Omega_\diamond^2\Psi_\theta$, contradicting the assumed decrease.

It also follows from lemma 4 that for public information to be welfare reducing, a higher reduction in the equilibrium Ω_* needs to be generated. The complementarities provide such an amplification mechanism. Consider, for instance, an increase in Ψ_θ. This causes ω to decrease, which always decreases the amount $\Omega^2\psi_\theta$ of private information generated by prices. Because $\partial H/\partial x > 0$, this prompts households to rely less on their private forecast, that is, to lower ω, which decreases $\Omega^2\psi_\theta$ further, prompts households to lower ω, and so on. Our results reveal that this effect alone can reduce the total amount $\Omega^2\psi_\theta + \Omega^2\Psi_\theta$ of information generated by prices, even if Ψ_θ increases.

VII. Robustness and Extensions

In this section we study two extensions of our basic model. In the first, we show that our results continue to hold in a cashless limit where, as in standard neo-Keynesian models, households hold no money balances. In the second, we show that our results continue to hold when complementarities in production give workers a motive to coordinate their labor supply decisions.

A. Cashless Limit

In this subsection, we show that our results remain valid in a "cashless limit" where the utility value of holding real money balances goes to zero at the same time that the money supply goes to zero. Our goal here is to show that our results are robust to a cashless limit resembling those commonly taken in recent monetary models and that our welfare results are not driven by household expected utility for real balances, which we do not derive from explicit micro foundations. Our main result in this section is as follows.

PROPOSITION 5 (Cashless limit). Suppose that velocity is equal to $\hat{V} = V/\Phi$ and that the aggregate money supply is equal to $\hat{M} = M\Phi$, for some $\Phi > 0$. Then

1. equilibrium prices and quantities are independent of Φ,
2. welfare depends on Φ but the optimal communication policy does not, and
3. as Φ goes to zero, the household's utility over money balances goes to zero, and the welfare impact of public communication stays bounded away from zero.

The first point of the proposition follows from the fact that equilibrium prices and quantities depend on nominal variables only through the product $\hat{M}\hat{V}$, which is kept constant and equal to MV. Intuitively, although the monetary base, M, goes to zero, velocity increases so that the amount of liquidity available for shopping stays the same. The second point holds because, with our log utility specification, proposition 2 holds for all Φ: welfare depends only on public information through an increasing function of the posterior precision about v. The third point follows because, in equilibrium the utility over real money balance is

$$\frac{\Phi}{V}\log\left(\frac{\Phi M}{P_t}\right) = \frac{\Phi}{V}\log\Phi + \frac{\Phi}{V}\log\left(\frac{M}{P_t}\right),$$

which goes to zero as Φ goes to zero. Welfare is a function only of expected consumption and of expected disutility of labor, which from point 1 remain affected by public communication.

B. *Information Aggregation versus Coordination*

In our basic model we showed that a central bank finds it optimal to release all its information or not to release any at all. Indeed, when it makes an information release, workers rationally coordinate their actions on this newly released public information, reducing the informativeness of nominal prices. Hellwig (2005) considered a neo-Keynesian setting with dispersed information and complementarities in production but no learning from prices. He showed that the coordination effect of public information is always socially beneficial. Thus, one may guess that when complementarities in production and learning from prices are combined, the central bank's optimal communication may become less extreme: perhaps the central bank would choose to release some, but not all, of its information.

Let us assume then that intermediate goods are complements in a CES final goods production function:

$$Y_t = \left[\int A_i Y_{it}^{(\phi-1)/\phi} di \right]^{\phi/(\phi-1)}, \tag{22}$$

with $\phi > 0$. In Appendix A we show that the fixed-point equation is the same as equation (13), but ψ_a needs to be replaced by $\psi_a/(\phi)^2$. After we make this substitution, the analysis of the fixed-point equation and the effects of information releases on total knowledge remain as in the previous sections. With regard to welfare, we now have the following proposition.[20]

PROPOSITION 6 (CES production). Suppose that the production function is given by equation (22) for some $\phi > 0$. Consider, in the cashless limit of proposition 5, the equilibrium that is most informative about velocity. Then, (1) proposition 3 continues to hold: the optimal communication policy of the central bank is bang-bang. (2) Proposition 4 has to be modified: consider any given finite increase of public precisions. Then there exists (ψ_a, ψ_θ) such that this increase is welfare decreasing if and only if $\delta > \phi$.

Away from the cashless limit and when $\phi \geq 1$, the $\delta > \phi$ condition

[20] Even though welfare is no longer separable in the CES case, the decentralized economy uses information efficiently when taking the information sets of workers as given. From Angeletos and Pavan (2007), this implies that more public and private information would always be welfare enhancing given an *exogenous information structure*. Proposition 6 shows that this is not the case when the information structure is endogenous.

for a welfare-reducing communication is sufficient but no longer necessary.

Proposition 6 shows that adding complementarities in production (which generate a social benefit from coordination) does not affect our bang-bang communication result. However, it makes it less likely that public information reduces welfare: when $\phi > 1$, it requires a higher elasticity of labor supply. Conversely, if $\phi < 1$ (so that there are social costs from coordination), then public information is more likely to reduce welfare.

VIII. Conclusions

We have studied a model in the spirit of Lucas (1972) in which agents remain uncertain about the nature of the shocks affecting the economy, even though they observe all prices in the economy and learn from them. We have characterized the conditions under which public announcements about real and nominal aggregate shocks reduce the informativeness of prices and actually may increase uncertainty about fundamentals and lower welfare. While our model is essentially static, techniques similar to those developed here may prove useful in studying the dynamic effects of information releases and in answering the timing question when to make public announcements. This is all left for future research.

Appendix A

Proofs

In the proof we explicitly distinguish between the prior information of the household about v and θ and the public information about v and θ provided by the central bank. We assume that the household starts from a prior that v and θ are normally distributed, are independent from each other and from everything else, and have prior means of zero and prior precisions $\bar{\Psi}_v$ and $\bar{\Psi}_\theta$. Before any market opens, the central bank provides two public signals:

$$z_v = v + \eta_v, \quad z_\theta = \theta + \eta_\theta,$$

where η_v and η_θ are normally distributed with mean zero and respective precision $\Psi_v - \overline{\Psi}_v$ and $\Psi_\theta - \overline{\Psi}_\theta$ and are independent from each other and from everything else. Thus, after the signals are observed but before markets open, the precisions of the household's information about v and θ are Ψ_v and Ψ_θ. Finally, the definition of an equilibrium has to be slightly altered: K_0 and k_0 are not constant but, instead, are affine functions of the public signals z_v and z_θ.

Also, we write all the proofs for a generalized version of our model, with a CRRA utility over consumption and a CES final good production function. That is, we assume that the intertemporal utility of the family is

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}+\frac{1}{V}\log\frac{M_{t-1}^d}{P_t}-\frac{\delta}{1+\delta}\int_0^1 L_{it}^{1+1/\delta}di\right)\right]$$

for $\gamma \geq 0$, where $\gamma = 1$ is the logarithmic utility specification. The final good production function is

$$Y_t = \left[\int_0^1 A_i Y_{it}^{(\phi-1)/\phi}di\right]^{\phi/(\phi-1)}$$

for $\phi \geq 0$, where $\phi = 1$ is the Cobb-Douglas specification. The definition of an equilibrium is the same as in the main text of the paper. Next we obtain the shopper's first-order conditions with respect to consumption and real money balances:

$$\lambda_t = C_t^{-\gamma},$$

$$\frac{\lambda_t}{P_t} = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] + \beta\frac{1}{MV},$$

where we assume, as we did in the text, that the shopper makes his decision under full information about aggregate shocks. The next step is to show that in an equilibrium, $\lambda_t/P_t = \beta/(1-\beta)/(MV)$.

A. Ruling Out Implosive or Explosive Solution

The proof follows directly from the results of Obstfeld and Rogoff (1983). We provide a direct proof here as our functional assumptions make it quite simple. For any $t \geq 1$, agents have full information regarding the state of the economy, and the first-order conditions with respect to money holdings imply that

$$\frac{\lambda_t}{P_t} = \frac{\beta}{MV} + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = \sum_{j=1}^{k}\frac{\beta^j}{MV} + \beta^k\mathbb{E}\left[\frac{\lambda_{t+k}}{P_{t+k}}\right], \tag{A1}$$

where we have used that, in equilibrium, money holdings are $M_t^d = M$. Note as well that

$$\lim_{k\to\infty}\beta^k\mathbb{E}_t\left[\frac{\lambda_{t+k}}{P_{t+k}}\right] \geq 0$$

since multipliers and prices are all nonnegative. Now, suppose that this last inequality is strict for some state at time t. Then, taking the limit in equation (A1) would imply that at that state,

$$\frac{\lambda_t}{P_t} > \frac{\beta}{(1-\beta)MV}. \tag{A2}$$

Consider now the following deviation: the consumer increases his consumption at this state at time t by ε and reduces his money holdings in all subsequent periods by ε/P_t. Such a deviation is budget feasible since the consumer is holding strictly positive amounts of money in any equilibrium path, as M is constant and $M > 0$.

The marginal impact of such a deviation on the present discounted value at time t is

$$C_t^{-\gamma}\varepsilon - \sum_{k>1}\beta^k \mathbb{E}_t\left[\frac{1}{MV}\varepsilon P_t\right] = \left[C_t^{-\gamma} - \frac{\beta}{(1-\beta)MV}P_t\right]\varepsilon.$$

Now we know that $\lambda_t = C_t^{-\gamma}$ from the first-order condition with respect to consumption. And thus, if (A2) holds, this feasible deviation increases the household's utility, which would contradict optimality. This implies that in any equilibrium

$$\lim_{k\to\infty}\beta^k \mathbb{E}_t\left[\frac{\lambda_{t+k}}{P_{t+k}}\right] = 0$$

for any state at time t, and thus

$$\frac{\lambda_t}{P_t} = \frac{\beta}{(1-\beta)MV} \tag{A3}$$

for all $t \geq 1$.

B. *The Equilibrium Equations in the CRRA/CES Model*

We start by deriving the equilibrium equations and proceed with some elementary algebraic manipulations.

1. Main Equations

We first derive the equilibrium equations in the generalized model. The final good producer's first-order condition is

$$P_t A_i \left(\frac{Y_t}{Y_{it}}\right)^{1/\phi} = P_{it},$$

and the intermediate good worker's first-order condition yields that intermediate good demand is the same as in the text:

$$Y_{it} = \Theta_i \left(\frac{\beta}{1-\beta}\mathbb{E}_{it}\left[\frac{1}{MV}\right]\Theta_i P_{it}\right)^{\delta}.$$

Next, we make the same normalization as in the text, $e^{\mu_v}M(1-\beta)/\beta = 1$, focus on time 0, and drop the time subscript to simplify notation. After taking logarithms, we obtain the set of equilibrium equations in log-linear form:

$$\text{quantity equation:} \quad \gamma y = v - p, \tag{A4}$$

$$\text{intermediate goods supply:} \quad y_i = (1+\delta)\theta_i + \delta\left(p_i - \mathbb{E}_i[v] + \frac{V_i[v]}{2}\right), \tag{A5}$$

$$\text{intermediate goods demand:} \quad y_i = y + \phi\left(a_i - \frac{1}{2\psi_a} + p - p_i\right), \tag{A6}$$

$$\text{aggregate output:} \quad y = \int_0^1 y_i di + \frac{\phi}{2(\phi - 1)}\Xi, \tag{A7}$$

where

$$\Xi = -\frac{1}{\psi_a} + \text{disp}\left(a_i + \frac{\phi - 1}{\phi} y_i\right),$$

and, for any variable z_i,

$$\text{disp}(z_i) \equiv \int_0^1 z_i^2 di - \left(\int_0^1 z_i di\right)^2$$

is the cross-sectional dispersion. The definition of a symmetric linear equilibrium is the same as in the text.

Note that, after integrating equation (A6) and plugging in the expression for y given by equation (A7), we get that

$$p = \int_0^1 p_i di + \frac{1}{2}\left(\frac{1}{\psi_a} - \frac{\Xi}{\varphi - 1}\right),$$

which proves lemma 1 for the CRRA-CES case.

2. Obtaining the Price Equations

Solving out for p using the system of equations (A4)–(A7) while first integrating (A5) and (A6), one obtains that

$$p = \frac{1}{1 + \gamma\delta}\left[v + \gamma\delta \int_0^1 \mathbb{E}_i[v] di - \gamma(1 + \delta)\theta - \frac{\gamma(\delta + \phi)}{2(\phi - 1)}\Xi + \frac{\gamma\delta}{2}\left(\frac{1}{\psi_a} - V_i[v]\right)\right]. \tag{A8}$$

Now, using equations (A5) and (A6) to solve out for p_i, together with $\gamma y = v - p$ and the price found above, one obtains an equation for the price in sector i:

$$p_i = \frac{1}{1 + \gamma\delta} v + \frac{\delta}{\delta + \phi}\mathbb{E}_i[v] - \frac{\delta(1 - \gamma\phi)}{(1 + \gamma\delta)(\delta + \phi)} \int_0^1 \mathbb{E}_j[v] dj - \frac{\gamma\delta}{2(1 + \gamma\delta)} V_i[v]$$
$$+ \frac{(1 + \delta)(1 - \gamma\phi)}{(1 + \gamma\delta)(\delta + \phi)}\theta - \frac{1 + \delta}{\delta + \phi}\theta_i + \frac{\phi}{\delta + \phi} a_i - \frac{1}{2(1 + \gamma\delta)} \frac{1}{\psi_a} \tag{A9}$$
$$+ \frac{1 - \gamma\phi}{2(1 + \gamma\delta)(\phi - 1)}\Xi.$$

C. *Proof of Proposition 1*

Letting K_0 and k_0 be affine functions of the announcements, z_v and z_θ, we can write the equilibrium prices as

$$p = K_0 + K_v(v - \theta/\Omega), \tag{A10}$$

$$p_i = k_0 + K_v(v - \theta/\Omega) + k_a(a_i - k_\varepsilon \varepsilon_i/\Omega), \tag{A11}$$

as long as K_v and k_a are not equal to zero. But in any linear equilibrium this must be the case. Suppose not and that $k_a = 0$. Then $\mathbb{E}_i[v|p, \{p_j\}_{j\in[0,1]}, \theta_i, z_v, z_\theta]$ would be independent of a_i; but according to (A9), the local price of intermediate i would be a linear function of a_i contradicting that $k_a = 0$. Similarly, if $K_v = 0$, it would follow that the price signals are not informative about v and so that $\mathbb{E}_i[v|p, \{p_j\}_{j\in[0,1]}, \theta_i, z_v, z_\theta]$ would just be a function of z_v. But then again, using (A9) implies that the resulting price will be a function of both z_v and v, which is a contradiction of $K_v = 0$.

1. Step 1: the Velocity Forecast, Conditional on the Prices

From equations (A10) and (A11), it follows that the observation of $\{p, \{p_j\}_{j\in[0,1]}, \theta_i, z_v, z_\theta\}$ is equivalent to observing $\{v - \theta/\Omega, a_i - (k_\varepsilon/\Omega)\varepsilon_i, \theta + \varepsilon_i, v + \eta_v, \theta + \eta_\theta\}$. One can transform the worker information set into conditionally independent signals centered around v: we can first replace $\theta + \varepsilon_i$ by $v - \theta/\Omega + (\theta + \varepsilon_i)/\Omega = v + \varepsilon_i/\Omega$ and then $a_i - k_\varepsilon \varepsilon_i/\Omega$ by $a_i - k_\varepsilon\varepsilon_i/\Omega + k_\varepsilon(v + \varepsilon_i/\Omega) = v + a_i/k_\varepsilon$. We obtain then

$$\begin{aligned}\mathbb{E}_i[v] &= \mathbb{E}[v|v - \theta/\Omega, v + \varepsilon_i/\Omega, v + a_i/k_\varepsilon, v + \eta_v, v + \eta_\theta/\Omega] \\ &= \frac{1}{\Psi}[\overline{\Psi}_\theta\Omega^2(v - \theta/\Omega) + \psi_\theta\Omega^2(v + \varepsilon_i/\Omega) + \psi_a k_\varepsilon^2(v + a_i/k_\varepsilon) \\ &\quad + (\Psi_v - \overline{\Psi}_v)(v + \eta_v) \qquad \text{(A12)} \\ &\quad + (\Psi_\theta - \overline{\Psi}_\theta)\Omega^2(v + \eta_\theta/\Omega)], \qquad \text{(A13)}\end{aligned}$$

where $\Psi = (V_i[v])^{-1} = \psi_a k_\varepsilon^2 + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2$.

2. Step 2: The Coefficients of the Price Functions

When this expectation is substituted into equation (A9), it follows that the log price of intermediate i must equal

$$\begin{aligned}p_i = &\left[1 - \frac{\Psi_v}{(1+\gamma\delta)\Psi}\right]v - \frac{\gamma(1+\delta)}{1+\gamma\delta}\left(1 + \frac{\delta}{1+\delta}\frac{\Psi_\theta}{\Psi}\Omega\right)\theta + \frac{\phi\Psi + k_\varepsilon\delta\psi_a}{(\delta+\phi)\Psi}a_i \\ &+ \frac{\gamma\delta}{1+\gamma\delta}\frac{\Psi_v - \overline{\Psi}_v}{\Psi}z_v + \frac{\gamma\delta}{1+\gamma\delta}\frac{\Psi_\theta - \overline{\Psi}_\theta}{\Psi}\Omega z_\theta + \frac{\Omega\psi_\theta\delta - (1+\delta)\Psi}{(\delta+\phi)\Psi}\varepsilon_i \\ &- \frac{\gamma\delta}{2(1+\gamma\delta)}V_i[v] - \frac{1}{2(1+\gamma\delta)\psi_a} + \frac{1-\gamma\phi}{2(1+\gamma\delta)(\phi-1)}\Xi.\end{aligned}$$

With the assumed functional form for the intermediate good price, equation (A11), it has to be the case that

$$K_v = 1 - \frac{\delta\gamma}{1+\delta\gamma}\frac{\Psi_v}{\Psi}; \quad k_a = \frac{\phi}{\delta+\phi} + \frac{\delta}{\delta+\phi}\frac{\psi_a k_\varepsilon}{\Psi};$$

$$K_\theta = -\frac{\gamma(1+\delta)}{1+\delta\gamma} - \frac{\delta\gamma}{1+\delta\gamma}\frac{\Psi_\theta}{\Psi}\Omega; \quad k_\varepsilon = \Omega\frac{(1+\delta)\Psi - \delta\psi_\theta\Omega}{\phi\Psi + k_\varepsilon\delta\psi_a},$$

and that

$$k_0 = \frac{\delta\gamma}{1+\delta\gamma}\left(\frac{\Psi_v - \overline{\Psi}_v}{\Psi} z_v + \frac{\Psi_\theta - \overline{\Psi}_\theta}{\Psi}\Omega z_\theta\right) + \frac{1-\gamma\phi}{2(1+\delta\gamma)(\phi-1)}\Xi$$
$$- \frac{1}{1+\delta\gamma}\frac{1}{2\psi_a} - \frac{\delta\gamma}{1+\delta\gamma}\frac{V_i(v)}{2}.$$

Keeping in mind that $\Omega = -K_v/K_\theta$, we obtain

$$\Omega = \frac{1 - [\delta\gamma/(1+\delta\gamma)](\Psi_v/\Psi)}{[\gamma(1+\delta)/(1+\delta\gamma)] + [\delta\gamma/(1+\delta\gamma)](\Psi_\theta/\Psi)\,\Omega}$$

$$\Leftrightarrow \Omega = \frac{1}{\gamma(1+\delta)} + \frac{\delta}{1+\delta}\frac{\psi_a k_\varepsilon^2 + \psi_\theta \Omega^2}{\Psi},$$

after rearranging. Finally, replacing this last formula for Ω into the equation for k_ε yields $k_\varepsilon = 1/(\gamma\phi)$. This delivers the main fixed-point equation:

$$\Omega = \frac{1}{\gamma(1+\delta)} + \frac{\delta}{1+\delta}\frac{\psi_v + \psi_\theta\Omega^2}{\psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2}, \tag{A14}$$

where we let $\psi_v \equiv \psi_a/(\gamma\phi)^2$.

Doing the same exercise but this time using equation (A8) delivers the last coefficient of the price equations:

$$K_0 = \frac{\delta\gamma}{1+\delta\gamma}\left(\frac{\Psi_v - \overline{\Psi}_v}{\Psi} z_v + \frac{\Psi_\theta - \overline{\Psi}_\theta}{\Psi}\Omega z_\theta\right) - \frac{\gamma(\delta+\phi)}{2(1+\delta\gamma)(\phi-1)}\Xi$$
$$+ \frac{\delta\gamma}{1+\delta\gamma}\left[\frac{1}{2\psi_a} - \frac{V_i(v)}{2}\right]. \tag{A15}$$

3. Step 3: The Coefficient Ξ

We now need to calculate $\Xi = -1/\psi_a + \mathrm{disp}(a_i + [(\phi-1)/\phi]y_i)$. To do so, we note that equation (A6) implies that

$$y_i = \mathrm{CCS} + \phi(a_i - p_i) = \mathrm{CCS} + \phi\left(a_i - k_a a_i + \frac{k_a k_\varepsilon}{\Omega}\varepsilon_i\right),$$

where CCS is shorthand for terms that are constant in the cross section. Recall that

$$\frac{k_a k_\varepsilon}{\Omega} = \frac{1+\delta}{\delta+\phi} - \frac{\delta}{\delta+\phi}\frac{\psi_\theta\Omega}{\Psi}.$$

With this in mind, we obtain that

$$\Xi = -\frac{1}{\psi_a} + \mathrm{disp}\left(a_i\left[1 + (\phi-1)\frac{\delta}{\delta+\phi}\left(1 - \frac{\psi_a k_\varepsilon}{\Psi}\right)\right] + \varepsilon_i\frac{\delta(\phi-1)}{\delta+\phi}\left(\frac{1+\delta}{\delta} - \frac{\psi_\theta\Omega}{\Psi}\right)\right)$$
$$= -\frac{1}{\psi_a} + \frac{1}{\psi_a}\left[1 + \frac{\delta(\phi-1)}{\delta+\phi} - \frac{\delta(\phi-1)}{\delta+\phi}\frac{\psi_a k_\varepsilon}{\Psi}\right]^2 + \frac{1}{\psi_\theta}\frac{\delta^2(\phi-1)^2}{(\delta+\phi)^2}\left(\frac{1+\delta}{\delta} - \frac{\psi_\theta\Omega}{\Psi}\right)^2.$$

Developing the square terms and using that $k_\varepsilon = 1/(\gamma\phi)$ together with

$$\psi_v + \psi_\theta \Omega^2 = \Psi \frac{1+\delta}{\delta}\left[\Omega - \frac{1}{\gamma(1+\delta)}\right],$$

which follows from the fixed-point equation, one obtains that

$$\Xi = \frac{(\phi-1)\{\gamma^2(1+\delta)^2(\phi-1)\phi^2\psi_v + \delta[\delta+(2+\delta)\phi]\psi_\theta\}}{\gamma^2\phi^2(\delta+\phi)^2\psi_v\psi_\theta} - \frac{\delta(\phi-1)[1+2\delta+\phi+\gamma(1+\delta)(\phi-1)\Omega]}{\gamma(\delta+\phi)^2\Psi}.$$

4. Step 4: Finishing Up

Now using (A6) together with (A4), one obtains the value of y_i given the price of the intermediate good, p_i:

$$y_i = \frac{v-p}{\gamma} + \phi\left(v + a_i - \frac{1}{2\psi_a} + p - p_i\right). \tag{A16}$$

Finally, we let

$$\gamma y = v - p. \tag{A17}$$

We have thus shown that in a linear equilibrium, prices take the linear form stated in the proposition and Ω must solve the fixed-point equation (A14). Note that sufficiency also follows: given a solution to the fixed-point equation (A14), the prices and quantities obtained from (A10), (A11), (A16), and (A17) form a linear equilibrium. The existence result follows by noticing that the fixed-point equation (A14) is a continuous mapping from [0, 1] into [0, 1], and thus there must be at least one fixed point.

Note that using that $\gamma y = v - p$, we get that

$$\begin{aligned} y = {} & \frac{\delta}{1+\gamma\delta}\frac{\overline{\Psi}_v}{\Psi}v + \left(\frac{1+\delta}{1+\gamma\delta} + \frac{\delta}{1+\gamma\delta}\frac{\Omega\overline{\Psi}_\theta}{\Psi}\right)\theta - \delta\frac{\Psi_\theta - \overline{\Psi}_\theta}{(1+\gamma\delta)\Psi}\Omega\eta_\theta \\ & - \delta\frac{\Psi_v - \overline{\Psi}_v}{(1+\gamma\delta)\Psi}\eta_v + \frac{\delta+\phi}{2(\phi-1)(1+\gamma\delta)}\Xi + \frac{\delta}{2(1+\gamma\delta)}\left(\frac{1}{\Psi} - \frac{1}{\psi_a}\right). \end{aligned} \tag{A18}$$

5. The Coefficients K_0 and k_0 in the Special Case in Which $\phi = \gamma = 1$

To recover the coefficient in the special case studied in the main text of the paper, we let ϕ and γ go to one. First, we note that, as ϕ goes to one,

$$\frac{\Xi}{\phi-1} \to \frac{2\delta(\Psi - \gamma\psi_v)}{\gamma^2(1+\delta)\Psi\psi_v}.$$

Replacing this into equation (A15), one gets that

$$K_0 \to \frac{\delta}{1+\delta}\left(\frac{\Psi_v - \overline{\Psi}_v}{\Psi}z_v + \frac{\Psi_\theta - \overline{\Psi}_\theta}{\Psi}\Omega z_\theta\right) + \frac{\delta}{2(1+\delta)}\left(\frac{1}{\Psi} - \frac{1}{\psi_a}\right),$$

and using that $\Xi \to 0$ as $\phi \to 1$, we get that

$$k_0 \to \frac{\delta}{1+\delta}\left(\frac{\Psi_v - \overline{\Psi}_v}{\Psi} z_v + \frac{\Psi_\theta - \overline{\Psi}_\theta}{\Psi}\Omega z_\theta\right) - \frac{1}{2(1+\delta)}\left(\frac{1}{\psi_a} + \frac{\delta}{\Psi}\right).$$

Letting $\overline{\Psi}_\theta = \Psi_\theta$ and $\overline{\Psi}_v = \Psi_v$ delivers the coefficients in the main text of the paper.

D. *Proof of Proposition 2*

The ex ante time 0 utility of a representative household is

$$\mathbb{E}\left[\frac{Y^{1-\gamma}-1}{1-\gamma} + \frac{1}{V}\log\left(\frac{M}{P_0}\right) - \frac{1+\delta}{\delta}\int_0^1 L_i^{\delta/(1+\delta)} di\right].$$

1. The Expected Utility of Consuming Aggregate Output

Given log normality, the first term can be written as

$$\frac{e^{(1-\gamma)w}-1}{1-\gamma},$$

where

$$w = \mathbb{E}[y] + \frac{1-\gamma}{2}V(y)$$

is the log certainty equivalent of aggregate output.

The ex ante expectation of log output.—From equation (A18) it follows that

$$\begin{aligned}\mathbb{E}[y] &= \frac{\delta+\phi}{2(\phi-1)(1+\gamma\delta)}\Xi + \frac{\delta}{2(1+\gamma\delta)}\left(\frac{1}{\Psi} - \frac{1}{\psi_a}\right)\\ &= E_0 - \frac{1+\delta+(\delta+\phi)(1-\gamma)+\gamma(1+\delta)(\phi-1)\Omega}{2\gamma(1+\gamma\delta)(\delta+\phi)}\frac{\delta}{\Psi},\end{aligned}$$

where

$$E_0 = \frac{\delta(1+\delta)}{2\gamma^2(1+\gamma\delta)\phi(\delta+\phi)}\frac{1}{\psi_v} + \frac{(1+\delta)^2(\phi-1)}{2(1+\gamma\delta)(\delta+\phi)}\frac{1}{\psi_\theta}.$$

The ex ante variance of log output.—Using (A18), one can compute the variance of log output, and we have that

$$V(y) = \frac{(1+\delta)^2}{(1+\gamma\delta)^2\overline{\Psi}_\theta} + \frac{2\delta(1+\delta)\Omega}{(1+\gamma\delta)^2}\frac{1}{\Psi} + \frac{\delta^2(\Psi_v + \Psi_\theta\Omega^2)}{(1+\gamma\delta)^2}\frac{1}{\Psi^2}.$$

Now we note that

$$\frac{\Psi_v + \Psi_\theta\Omega^2}{\Psi} = 1 - \frac{\psi_v + \psi_\theta\Omega^2}{\Psi} = \frac{1+\delta}{\delta}\left[\Omega - \frac{1}{\gamma(1+\delta)}\right]$$

from the fixed-point equation, and thus plugging back into the variance of log output, we obtain

$$V(y) = \frac{(1+\delta)^2}{(1+\gamma\delta)^2\overline{\Psi}_\theta} + \frac{\delta[1+\gamma\delta+\gamma(1+\delta)\Omega]}{\gamma(1+\gamma\delta)^2\Psi}.$$

The log certainty equivalent of aggregate output.—Taken together, these formulas imply that the log certainty equivalent of aggregate output is

$$w = \mathbb{E}[y] + \frac{1-\gamma}{2}V(y)$$
$$= E_0 + \frac{(1-\gamma)(1+\delta)^2}{2(1+\gamma\delta)^2\overline{\Psi}_\theta} - \frac{\delta(1+\delta)[1+\gamma\delta+\gamma(1+\delta)(\gamma\phi-1)\Omega]}{2\gamma(1+\gamma\delta)^2(\delta+\phi)\Psi}. \tag{A19}$$

2. The Expected Utility of Real Balances

To find the expected utility of real balances, we recall that, from Section A, $\lambda/P = \beta/[(1-\beta)MV]$ and $\lambda = Y^{-\gamma}$. Therefore,

$$\frac{M}{P} = \frac{\beta}{1-\beta}\frac{Y^\gamma}{V}.$$

This yields a utility over real balances of

$$\frac{1}{V}\log\frac{M}{P} = -\frac{1}{V}\log V + \frac{1}{V}\log\frac{\beta}{1-\beta} + \frac{\gamma}{V}\log Y.$$

Note that the ex ante expectation of the first two terms is independent of the communication policy of the central bank. The only thing that matters for the welfare impact of the communication policy is the expectation of the third term:

$$\mathbb{E}\left[\frac{1}{V}\log Y\right] = e^{-\mu_v}\mathbb{E}[e^{-v}y] = e^{-\mu_v}\mathbb{E}[e^{-v}][\mathbb{E}[y] - \mathrm{Cov}(v, y)]$$
$$= e^{-\mu_v+1/2\bar{\Psi}_v}\left(\mathbb{E}[y] - \frac{\delta}{1+\delta\gamma}\frac{1}{\Psi}\right),$$

where the last equality follows after noting that equation (A18) implies that $\mathrm{Cov}(v, y) = \delta/[(1+\delta\gamma)\Psi]$. Taken together, we obtain that the expected utility over real money balances

$$\mathbb{E}\left[\frac{1}{V}\log\frac{M}{P}\right] = RB_0 - e^{-\mu_v+1/2\bar{\Psi}_v}\left\{\frac{\delta[1+(2+\gamma)\delta+\phi(1+\gamma)+\gamma(1+\delta)(\phi-1)\Omega]}{2(1+\gamma\delta)(\delta+\phi)}\right\}\frac{1}{\Psi},$$

where

$$RB_0 = \mathbb{E}\left[-\frac{1}{V}\log V + \frac{1}{V}\log\frac{\beta}{1-\beta}\right] + e^{-\mu_v+1/2\bar{\Psi}_v}\gamma E_0.$$

3. The Expected Cost of Supplying Labor

To calculate the expected cost of supplying labor, we start by evaluating

$$\mathbb{E}\left[\int_0^1 L_i^{1+1/\delta} di\right]$$

$$= \mathbb{E}[\mathbb{E}[L_i^{1+1/\delta}|v, \theta]] = \mathbb{E}[L_i^{1+1/\delta}] = \mathbb{E}\left[L_i \mathbb{E}_i\left[\lambda \frac{P_i}{P}\Theta_i\right]\right] \tag{A20}$$

$$= \mathbb{E}\left[L_i \lambda \frac{P_i}{P}\Theta_i\right] = \mathbb{E}\left[Y_i Y^{-\gamma}\frac{\partial Y}{\partial Y_i}\right] = \mathbb{E}\left[Y^{-\gamma}\mathbb{E}\left[Y_i \frac{\partial Y}{\partial Y_i}\middle| v, \theta\right]\right] \tag{A21}$$

$$= \mathbb{E}\left[Y^{-\gamma}\int_0^1 Y_i \frac{\partial Y}{\partial Y_i} di\right] = \mathbb{E}[Y^{1-\gamma}]. \tag{A22}$$

In (A20) the first equality follows from the law of large numbers: the average labor cost is equal to the expected labor cost, conditional on the realization of the aggregate state, (v, θ). The second equality follows from the law of iterated expectations. The third equality follows from the worker's first-order condition. In (A21), the first equality follows from the law of iterated expectations. The second equality follows because workers observe Θ_i and $Y_i = \Theta_i L_i$, because $\lambda = Y^{-\gamma}$, and because, from the final good firm first-order condition, $P_i/P = \partial Y/\partial Y_i$. The third inequality follows from an application of the law of iterated expectations. In (A22), the first equality follows from the law of large numbers, just as in equation (A20). The second equality follows because the production function has constant returns to scale, and hence from Euler's theorem for homogeneous functions, we have that $\int_0^1 Y_i \partial Y/\partial Y_i di = Y$. Thus, the expected cost of supplying labor can be written

$$\frac{\delta}{1+\delta}e^{(1-\gamma)w},$$

where w is the log certainty equivalent of aggregate output, which we calculated before.

4. Welfare with Log Utility and a Cobb-Douglas Production Function

In this case, where $\phi = \gamma = 1$, the expected cost of supplying labor is zero and welfare reduces to

$$\underbrace{E_0 - \frac{\delta}{2(1+\delta)}\frac{1}{\Psi}}_{\text{expected log output}} + \underbrace{RB_0 - \frac{\delta e^{-\mu_v + 1/2\bar{\Psi}_v}}{2(1+\delta)}\frac{1}{\Psi}}_{\text{expected utility from real balances}}.$$

This is an increasing function of Ψ, which establishes proposition 2.

5. Welfare in the Cashless Limit with CRRA Utility and a CES Production Function

In this case we can ignore the expected utility over real balances and focus on the sum of the expected utility of consuming aggregate output and the expected cost of supplying labor:

$$\underbrace{\frac{e^{(1-\gamma)w}-1}{1-\gamma}}_{\text{expected utility of output}} - \underbrace{\frac{\delta(1-\gamma)}{1+\delta}e^{(1-\gamma)w}}_{\text{expected cost of labor}} = \frac{(\delta+\gamma)e^{(1-\gamma)w}-(1+\delta)}{(1-\gamma)(1+\delta)}.$$

which is an increasing function of the log certainty equivalent of aggregate output, w, and hence, by equation (A19), an increasing function of Ψ.

E. *Proof of Lemma 3*

Consider the fixed-point equation (A14):

$$\Omega = \frac{1}{\gamma(1+\delta)} + \frac{\delta}{1+\delta}\frac{\psi_v + \psi_\theta\Omega^2}{\psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2} \equiv F(\Omega, \Psi_v, \Psi_\theta).$$

Clearly, when Ω is large enough, the left-hand side of the fixed-point equation is larger than the right-hand side. Therefore, the highest solution, Ω_*, of the fixed-point equation solves

$$\Omega_* = \max\{\Omega : \Omega \leq F(\Omega, \Psi_v, \Psi_\theta)\}. \tag{A23}$$

Now consider two vectors $(\Psi_v^{(1)}, \Psi_\theta^{(1)})$ and $(\Psi_v^{(2)}, \Psi_\theta^{(2)})$ such that $(\Psi_v^{(1)}, \Psi_\theta^{(1)}) \leq (\Psi_v^{(2)}, \Psi_\theta^{(2)})$, by which we mean, as usual, that the inequality holds component per component with at least one strict inequality. Let $\Omega_*^{(1)}$ and $\Omega_*^{(2)}$ be the corresponding solutions of (A23). We have

$$\Omega_*^{(1)} = F(\Omega_*^{(1)}, \Psi_v^{(1)}, \Psi_\theta^{(1)}) > F(\Omega_*^{(1)}, \Psi_v^{(2)}, \Psi_\theta^{(2)}),$$

where the equality follows because $\Omega_*^{(1)}$ is a solution of the fixed-point equation when $\Psi_v = \Psi_v^{(1)}$ and $\Psi_\theta = \Psi_\theta^{(1)}$, and the inequality follows because $F(\Omega, \Psi_v, \Psi_\theta)$ is a strictly decreasing function of Ψ_v and Ψ_θ. But, from the characterization of $\Omega_*^{(2)}$ in (A23), it then follows that $\Omega_*^{(2)} < \Omega_*^{(1)}$. This establishes point i of the lemma.

Turning to point ii, consider a sequence of Ψ_v and/or Ψ_θ converging to infinity and the corresponding sequence of Ω_*. From the fixed-point equation, it follows that Ω_* is greater than $1/[\gamma(1+\delta)]$, the minimum of the right-hand side, and smaller than $(1+\delta\gamma)/[\gamma(1+\delta)]$, the maximum of the right-hand side. So the sequence of Ω_* remains in a compact set, and it must have at least one accumulation point, Ω_*^∞. By continuity of $F(\Omega, \Psi_v, \Psi_\theta)$, it follows that $\Omega_*^\infty = F(\Omega_*^\infty, \infty) = 1/[\gamma(1+\delta)]$. This shows that $1/[\gamma(1+\delta)]$ is the unique accumulation point of the sequence and consequently is also its limit. This establishes point ii.

An analogous argument shows point iii of the proposition.

F. *Proof of Proposition 3*

1. Preliminaries

A criterion for assessing the welfare impact of public information.—From Sections D.4 and D.5, we know that in both cases of interest (i.e., log utility plus Cobb-Douglas production and CRRA plus CES production plus cashless limit), ex ante welfare can be shown to be an increasing function of the log certainty equivalent of aggregate output, which we calculated in equation (A19). That equation then implies that increasing (Ψ_θ, Ψ_v) increases welfare if and only if it increases the following function:

$$-\frac{[(1+\delta\gamma)/(1+\delta)] + \gamma\Omega(\gamma\phi - 1)}{\Psi}. \tag{A24}$$

From the fixed-point equation (A14), it follows that

$$\frac{1}{\Psi} = \frac{\delta+1}{\delta\gamma}\frac{\gamma\Omega - [1/(1+\delta)]}{\psi_v + \psi_\theta\Omega^2}.$$

Plugging this back into (A14), we obtain that increasing (Ψ_v, Ψ_θ) increases welfare if and only if the resulting decrease in Ω increases the following function:

$$\begin{aligned}\mathcal{W}(\Omega) &\equiv -\frac{\{\gamma\Omega - [1/(1+\delta)]\}[(1+\delta\gamma)/(1+\delta) + \gamma\Omega(\gamma\phi-1)]}{\psi_v + \psi_\theta\Omega^2} \\ &= -\frac{N_2\Omega^2 + N_1\Omega + N_0}{\psi_v + \psi_\theta\Omega^2},\end{aligned} \tag{A25}$$

where

$$N_2 = \gamma^2(\gamma\phi - 1), \quad N_1 = \frac{\gamma(1+\delta\gamma)}{1+\delta} - \frac{\gamma(\gamma\phi-1)}{1+\delta}, \quad \text{and} \quad N_0 = -\frac{1+\delta\gamma}{(1+\delta)^2}.$$

But (Ψ_v, Ψ_θ) affects (A25) only through its impact on Ω. Moreover, Ω is strictly decreasing in (Ψ_v, Ψ_θ). So, to assess the welfare impact of public information, it will be sufficient to study how equation (A25) depends on Ω.

A cutoff weight for public information to be welfare improving.—The derivative of (A25) with respect to Ω is

$$\mathcal{W}(\Omega) = \frac{N_1\Omega^2 - 2[N_2(\psi_v/\psi_\theta) - N_0]\Omega - N_1(\psi_v/\psi_\theta)}{(\psi_v + \psi_\theta\Omega^2)^2/\psi_\theta} \equiv \frac{G(\Omega)}{(\psi_v + \psi_\theta\Omega^2)^2/\psi_\theta},$$

so the sign of the derivative depends on the sign of the second-order polynomial $G(\Omega)$. We first note that, from lemma 3, $\Omega \in [1/[\gamma(1+\delta)], (1+\delta\gamma)/[\gamma(1+\delta)]]$. Evaluating the function $G(\Omega)$ as the lower bound of this interval, we find, after some calculations,

$$G\left(\frac{1}{\gamma(1+\delta)}\right) = -\frac{\delta+\phi}{1+\delta}\left[\frac{1}{(1+\delta)^2} + \gamma^2\frac{\psi_v}{\psi_\theta}\right] < 0. \tag{A26}$$

There are thus three cases to consider.

Case 1: $N_1 < 0$. Then $G(0) = -\psi_v/\psi_\theta N_1 > 0$, and $G(\infty) = G(-\infty) = -\infty$. Thus, $G(\Omega)$ has two zeros, one strictly negative and one strictly positive. Moreover, from (A26), the positive zero must be smaller than $1/[\gamma(1+\delta)]$. Therefore, in equilibrium, $G(\Omega_*)$ is negative given that $\Omega_* \geq 1/[\gamma(1+\delta)]$, and thus $\mathcal{W}'(\Omega_*) < 0$. Given that Ω_* is a strictly decreasing function of public information, (Ψ_v, Ψ_θ), it follows that welfare is a strictly increasing function of (Ψ_v, Ψ_θ).

Case 2: $N_1 = 0$ and $G(\Omega)$ is an affine function. For N_1 to be equal to zero, it must be the case that $\gamma\phi > 1$, and so $N_2 > 0$. Since N_0 is negative, it follows that, in the function $G(\Omega)$, the coefficient on Ω is negative. So $G(\Omega)$ is decreasing and is negative at $\Omega = 1/[\gamma(1+\delta)]$. Then again, in equilibrium, $G(\Omega_*)$ is negative. Given that Ω is a strictly decreasing function of (Ψ_v, Ψ_θ), it follows that welfare is a strictly increasing function of public information, (Ψ_v, Ψ_θ).

Case 3: $N_1 > 0$. Then $G(0) < 0$, $G(\infty) = +\infty$, and $G(\Omega)$ is negative when $\Omega = 1/[\gamma(1+\delta)]$. Therefore, there exists a unique cutoff $\bar{\Omega} > 1/[\gamma(1+\delta)]$ such that $G(\Omega) < 0$ for $\Omega < \bar{\Omega}$ and $G(\Omega) > 0$ for $\Omega > \bar{\Omega}$.

2. Proof of Proposition 3

The above analysis shows that there exists some

$$\overline{\Omega} \in \left(\frac{1}{\gamma(1+\delta)}, \frac{1+\delta\gamma}{\gamma(1+\delta)}\right]$$

such that, for all

$$\Omega_* \in \left(\frac{1}{\gamma(1+\delta)}, \frac{1+\delta\gamma}{\gamma(1+\delta)}\right),$$

$$\Omega_* > \overline{\Omega} \Rightarrow G(\Omega_*) > 0,$$

$$\Omega_* < \overline{\Omega} \Rightarrow G(\Omega_*) < 0.$$

In particular, in cases 1 and 2, $\overline{\Omega} = (1+\delta\gamma)/[\gamma(1+\delta)]$ and $G(\Omega_*) < 0$ for all possible equilibrium values of Ω_*.

The above implies that the criterion $\mathcal{W}(\Omega)$ is U-shaped. It is strictly decreasing for $\Omega_* \in [1/[\gamma(1+\delta)], \overline{\Omega})$ and strictly increasing for $\Omega_* \in [\overline{\Omega}, (1+\delta\gamma)/[\gamma(1+\delta)]]$.

Now suppose that the government has independent signals about v and θ that would allow it to increase public information from $\Psi_v^{(0)}$ and $\Psi_\theta^{(0)}$ to $\Psi_v^{(1)} \geq \Psi_v^{(0)}$ and $\Psi_\theta^{(1)} \geq \Psi_\theta^{(0)}$, with at least one strict inequality. We want to show that, if it releases independent signals, the government's optimal policy is bang-bang: it is best for the government to release all or none of the information. To that end, consider any continuously increasing curve linking $(\Psi_v^{(0)}, \Psi_\theta^{(0)})$ to $(\Psi_v^{(1)}, \Psi_\theta^{(1)})$ in the (Ψ_v, Ψ_θ) space, parameterized by $x \in [0,1]$. Clearly, any partial release of independent information by the government will move the economy to a point lying on such a curve. After calculating the highest fixed point of the equilibrium equation (A14) for every $x \in [0,1]$, one obtains a strictly decreasing, possibly discontinuous, function $\Omega_*(x)$. Since $\mathcal{W}(\Omega)$ is U-shaped, it follows that

$$\max_{x\in[0,1]} \mathcal{W}(\Omega(x)) = \max_{\Omega\in[\Omega_*(0),\Omega_*(1)]} \mathcal{W}(\Omega) = \max\{\mathcal{W}(\Omega_*(0)), \mathcal{W}(\Omega_*(1))\}.$$

That is, welfare is maximized if the government releases all or none of the information.

G. *Proof of Proposition 4*

First part of the proposition.—We start first by showing that $\mathcal{W}(\Omega)$ is negative. To see this note that from equation (A25), $\mathcal{W}(\Omega)$ is negative if and only if

$$\frac{1+\delta\gamma}{1+\delta} + \gamma\Omega(\gamma\phi - 1) \geq 0. \tag{A27}$$

This is positive when $\gamma\phi \geq 1$. When $\gamma\phi < 1$, equation (A27) holds for all Ω if and only if it holds for $\Omega = (1+\delta\gamma)/[\gamma(1+\delta)]$, that is, if and only if

$$\frac{1+\delta\gamma}{1+\delta} + \frac{\gamma(1+\delta\gamma)}{\gamma(1+\delta)}(\gamma\phi - 1) = \frac{(1+\delta\gamma)\gamma\phi}{1+\delta} \geq 0,$$

a condition that is clearly satisfied.

Next, recall that when either Ψ_v or Ψ_θ goes to infinity, Ω converges to $1/[\gamma(1+\delta)] > 0$. Therefore, $\Psi = \psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2$ goes to infinity and $\mathcal{W}(\Omega)$ goes to zero. Given that $\mathcal{W}(\Omega)$ is negative, this implies that a sufficiently large increase in public information increases welfare.

Second part of the proposition: Sufficiency.—Suppose $\delta > \phi$. All we need to show is that, in this case, for any given finite increase in public information resulting in public precisions (Ψ_v, Ψ_θ), there exists some (ψ_v, ψ_θ) such that $G(\Omega_*) > 0$: indeed, if $G(\Omega_*) > 0$, then we know that the increase in public information has resulted in a decrease in Ω_* leaving us in a region where $\mathcal{W}(\Omega)$ is still increasing. Given that $\mathcal{W}(\Omega)$ is U-shaped, this implies that welfare has necessarily decreased.

First, we note that, as either ψ_θ or ψ_v goes to infinity, $\Omega_* \to (1+\delta\gamma)/[\gamma(1+\delta)]$. Indeed, from the fixed-point equation (A14),

$$0 \leq \frac{1+\delta\gamma}{\gamma(1+\delta)} - \Omega = \frac{\delta}{1+\delta}\frac{\Psi_v + \Psi_\theta\Omega^2}{\psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2}.$$

The result follows since Ω_* is bounded away from zero and infinity. Next, simple calculations show that

$$G\left(\frac{1+\delta\gamma}{\gamma(1+\delta)}\right) = G_0 + G_1\frac{\psi_v}{\psi_\theta}, \tag{A28}$$

where

$$G_0 = \frac{(1+\delta\gamma)^2}{(1+\delta)^3}(\delta - \phi)$$

and

$$G_1 = \frac{\gamma^2[\phi + \delta(-1 + 2\gamma\phi)]}{1+\delta}.$$

If $\delta > \phi$, then G_0 is positive, and thus equation (A28) implies that G is positive for small enough ψ_v/ψ_θ. Given the continuity of $G(\Omega)$ in both ψ_v/ψ_θ and Ω, we conclude that if ψ_v and ψ_θ are large enough but the ratio ψ_v/ψ_θ is small enough, then $G(\Omega_*)$ can be made positive.

Second part of the proposition: Necessity.—If $\delta \leq \phi$, then for any ψ_v and ψ_θ, we know that $G((1+\delta\gamma)/[\gamma(1+\delta)]) < 0$ as both $G_0 \leq 0$ and $G_1 < 0$. Given that we have already shown that $G(1/[\gamma(1+\delta)]) < 0$, it follows then that in this case, for all Ω_*, $G(\Omega_*) < 0$, and thus $\mathcal{W}(\Omega_*)$ is always decreasing in Ω_*. Since Ω_* is decreasing in public information, it follows that public announcements are always beneficial.

H. Ruling Out $k_\theta \neq 0$ or $K_\theta \neq 0$

To simplify the exposition, in Section III.A we assumed that, by observing velocity shock and the final good price, the shopper is able to infer the exact realization of θ from the observation of prices. In the class of symmetric linear equilibria, this amounts to assuming that either $K_\theta \neq 0$ or $k_\theta \neq 0$. In this section, we show that this property must, in fact, hold in any symmetric linear equilibrium.

LEMMA 5. In a symmetric linear equilibrium, $K_\theta \neq 0$ or $k_\theta \neq 0$.

To prove the lemma, we first note that, when the shopper is uncertain about what is on the right-hand side of his budget constraint, he faces the additional constraint that

$$C_0 \leq \min\left\{\frac{M^d_{-1}}{P_0} + Y_0\right\}, \tag{A29}$$

where the min is taken across all realizations of the state that have a nonzero probability, according to the posterior probability distribution of the shopper. This constraint says that the shopper cannot spend more than the minimum amount of resources he expects to receive by the end of the period—the intraperiod version of Aiyagari's (1994) natural borrowing limit. The first-order conditions of the shopper with respect to money balances and consumptions are then

$$\frac{\lambda_0}{P_0} = \beta\frac{\lambda_1}{P_1} + \beta\frac{1}{MV} = \frac{\beta}{1-\beta}\frac{1}{MV},$$

$$C_0^{-\gamma} \geq \frac{\lambda_0}{P_0},$$

with an equality if (A29) is not binding.

Now suppose that there exists a linear equilibrium such that $K_\theta = 0$ or $k_\theta = 0$. Then the intermediate good workers can perfectly infer v from the observation of nominal prices and face no uncertainty about λ_0. Therefore, the intermediate good supply is

$$y_i = (1+\delta)\theta_i + \delta(p_i - v).$$

Equating this with the intermediate good demand, we find that

$$(1+\delta)\theta_i + \delta(p_i - v) = y + \phi\left(a_i - \frac{1}{2\psi_a} + p - p_i\right).$$

Now recall that, because of equation (12), in a linear equilibrium p_i and p must have the same loading on v and θ, K_v, and K_θ. Averaging the above equality across sectors while keeping in mind our maintained assumption that $K_\theta = 0$, we find that, up to some constant,

$$(1+\delta)\theta + \delta(K_v - 1)v = y. \tag{A30}$$

Now let us go back to the shopper's problem. If, in a linear equilibrium, the shopper does not infer the exact realization of θ, then his posterior distribution is a nondegenerate normal, so it has an unbounded negative support. Thus equation (A30) implies that the minimum of Y_0 is zero and (A29) becomes the cash-in-advance (CIA) constraint:

$$C_0 \leq \frac{M^d_{-1}}{P_0}. \tag{A31}$$

Next, there are two cases to consider. If the CIA is not binding, then $1/C = \lambda$ and the quantity equation is

$$p = v - \gamma y = v(1 - \gamma\delta K_v + \gamma\delta) - \gamma(1 + \delta)\theta,$$

which implies that the price does depend on θ. The second case occurs if the CIA is binding, $p = m - y$, and, then again, p depends on θ. This contradicts the assumption that $K_\theta = 0$.

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