Foreign Reserve Management at Zero Interest Rates

M. Amador[1] J. Bianchi[2] L. Bocola[3] F. Perri[4]

[1]Minneapolis Fed and U of Minnesota
[2]Minneapolis Fed
[3]Northwestern U
[4]Minneapolis Fed

Exchange rates, and foreign reserves accumulation

- A Central Bank sets an exchange rate policy that makes domestic assets attractive.
  - $\Rightarrow$ Capital flows in
- Close to zero interest rates, the CB has a problem:
  - Domestic interest rates cannot fall to restore equilibrium
- One option:
  - Accumulate foreign assets and reverse the inflow
- In a world with limited arbitrage, this works
  - But may be costly

Our questions

1. How to invest the foreign reserves?
    - Doesn't matter, under perfect mobility (Backus Kehoe, 89)
    - Our answer: It does, under imperfect mobility

Our questions

1. How to invest the foreign reserves?
   - Doesn't matter, under perfect mobility (Backus Kehoe, 89)
   - Our answer: It does, under imperfect mobility

The CB has two goals:

1. Minimize the distortions imposed by ZLB
2. Minimize resource losses generated its FX interventions

- Goals not necessarily aligned
- Degree of openness controls the trade-off
- Different prescriptions for the assets that must be purchased

Our questions

2. What's the right measure of the losses?
    - Covered interest parity versus uncovered

3. Should foreign capital be restricted once inside borders?
    - In general, yes

Framework

- Two-period model (similar to *Backus-Kehoe, 89*)
    - Small open economy (government + households)
    - International Financial Market
    - International Arbitrageurs
- Time $t \in \{0, 1\}$
- Uncertainty realized at $t = 1$, $s \in S \equiv \{s_1, ..., s_N\}$
- Probability $\pi(s) \in (0, 1]$
- One good – no production
- Law of one price

Asset markets: complete but segmented

International financial markets (IFM)

- Full set of Arrow-Debreu (real) securities:
  - Security s: 1 unit of consumption good in state s, 0 otherwise
  - Price $q(s)$ in terms of goods at $t = 0$

Domestic financial market

- Money + full set of Arrow-Debreu (nominal) securities
  - Security s: 1 unit of money in state s, 0 otherwise
  - Price $p(s)$ in terms of money at $t = 0$

Arbitrageurs

- Can move resources to the SOE from IFM

Model: Small open economy

Households

- Endowment: $(y_0, \{y_1(s)\})$, transfers: $(T_0, \{T_1(s)\})$

$$\max_{c_0,\{c_1,a,f\},m} \left\{ u_0(c_0) + \beta \sum_{s\in S} \pi(s) u_1(c_1(s)) + h\left(\frac{m}{e_0}\right) \right\}$$

$$y_0 + T_0 = c_0 + \sum_{s\in S} \left[q(s) f(s) + p(s) \frac{a(s)}{e_0} \right] + \frac{m}{e_0}$$

$$y_1(s) + T_1(s) + f(s) + \frac{a(s) + m}{e_1(s)} = c_1(s) \quad \forall s \in S$$

$$f(s) \geq 0 \quad \forall s \in S$$

e_0, $e_1(s)$: exchange rates at $t = 0$ and $t = 1$

$f(s), a(s)$: holdings of foreign and domestic security s

m: money holdings, $\overline{x}$: satiation point of h

Model: Small open economy

Government

- Exchange rates $\{e_0, e_1(s)\}$; money, M; amounts invested at home and abroad, $A(s)$ and $F(s)$; and transfers $(T_0, \{T_1(s)\})$.

- Budget constraint:

$$\sum_s p(s)\frac{A(s)}{e_0} + \sum_s q(s)F(s) + T_0 = \frac{M}{e_0}$$

$$\frac{M}{e_1(s)} + T_1(s) = \frac{A(s)}{e_1(s)} + F(s) \quad \forall s \in S$$

- In addition (only because of symmetry): $F(s) \geq 0 \quad \forall s \in S$

  Describe gov't objective later on

Model: Foreign arbitrageurs

- Endowed with total resources $\overline{w}$

$$\max_{m^\star,\{a^\star,f^\star\}} \sum_{s\in S} q(s)c^\star(s)$$

subject to:

$$\overline{w} = \sum_{s\in S} p(s)\frac{a^\star(s)}{e_0} + \frac{m^\star}{e_0} + \sum_{s\in S} q(s)f^\star(s)$$

$$c^\star(s) = \frac{a^\star(s) + m^\star}{e_1(s)} + f^\star(s)$$

$$a^\star(s) \geq 0, m^\star \geq 0, f^\star(s) \geq 0$$

NOTE: It is the ability of arbitrageurs to buy domestic securities that allows the SOE to borrow internationally

Equilibrium definition

Take a given $(e_0, \{e_1(s)\})$

Equilibrium

HH's consumption, $(c_0, \{c_1(s)\})$, and asset positions, $(\{a(s), f(s)\}, m)$; arbitrageurs consumption, $\{c^\star(s)\}$, and asset positions $(\{a^\star(s), f^\star(s)\}, m^\star)$; government transfers $(T_0, \{T_1(s)\})$, asset $(\{A(s), F(s)\})$ and liabilities M; such that

1. HH and arbitrageurs maximize taking prices as given,
2. the government budget constraint holds, and
3. the domestic financial markets clear:

$$m + m^\star = M$$

$$a(s) + a^\star(s) + A(s) = 0 \quad \forall s \in S$$

Government objective

- Government desires to implement $(e_0, \{e_1(s)\})$ – this is given.
- Chooses policy $(M, \{A(s), F(s)\})$ and $(T_0, \{T_1(s)\})$ as to implement the equilibrium that maximizes household welfare.
    - *optimal equilibrium / optimal equilibrium allocation*.

Government objective

- Government desires to implement $(e_0, \{e_1(s)\})$ – this is given.
- Chooses policy $(M, \{A(s), F(s)\})$ and $(T_0, \{T_1(s)\})$ as to implement the equilibrium that maximizes household welfare.
  - *optimal equilibrium / optimal equilibrium allocation*.
- For the rest of the talk: no income in the second period:

$$y_1(s) = 0 \text{ for all } s$$

Characterizing monetary equilibria: Arbitrage returns

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_0}{e_1 p(s)}}{\frac{1}{q(s)}} - 1$$

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_0}{e_1 p(s)}}{\frac{1}{q(s)}} - 1$$

- Using the HH's FOC

$$u_0'(c_0) = \beta\pi(s)\frac{e_0}{e_1(s)p(s)}u_1'(c_1(s))$$

Characterizing monetary equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_0}{e_1 p(s)}}{\frac{1}{q(s)}} - 1$$

- Using the HH's FOC

$$u_0'(c_0) = \beta\pi(s)\frac{e_0}{e_1(s)p(s)}u_1'(c_1(s))$$

we get

$$\kappa(s) = \frac{q(s)u_0'(c_0)}{\beta\pi(s)u_1'(c_1(s))} - 1$$

In any equilibrium $\kappa(s) \geq 0$ and $f(s) = 0$ if strict

Characterizing monetary equilibria: Resource constraint

Using the HH and government budget constraints, together with market clearing,

$$y_0 - c_0 - \sum_{s \in S} q(s) c_1(s)$$

Characterizing monetary equilibria: Resource constraint

Using the HH and government budget constraints, together with market clearing,

$$y_0 - c_0 - \sum_{s\in S} q(s)c_1(s) - \underbrace{\left(\sum_{s\in S}\kappa(s)\frac{p(s)a^\star(s)}{e_0} + \left[\sum_{s\in S}\frac{q(s)e_0}{e_1(s)} - 1\right]\frac{m^\star}{e_0}\right)}_{\text{potential "arbitrage losses"}\equiv L} = 0$$

Characterizing monetary equilibria: Arbitrageurs optimization

Arbitrageurs problem $\Leftrightarrow$ maximize "arbitrage losses" L

$$L = \max_{m^\star, \{a^\star(s)\}} \left\{ \sum_{s \in S} \kappa(s) \frac{p(s) a^\star(s)}{e_0} + \underbrace{\left[\sum_{s \in S} \frac{q(s) e_0}{e_1(s)} - 1 \right]}_{\kappa^m} \frac{m^\star}{e_0} \right\}$$

$$\text{s.t.} \quad \frac{m^\star}{e_0} + \sum \frac{p(s) a^\star(s)}{e_0} \leq \overline{w}; \qquad m^\star \geq 0, a^\star(s) \geq 0 \ \forall s$$

Characterizing monetary equilibria: Arbitrageurs optimization

Arbitrageurs problem $\Leftrightarrow$ maximize "arbitrage losses" L

$$L = \max_{m^\star, \{a^\star(s)\}} \left\{ \sum_{s \in S} \kappa(s) \frac{p(s)a^\star(s)}{e_0} + \underbrace{\left[\sum_{s \in S} \frac{q(s)e_0}{e_1(s)} - 1 \right]}_{\kappa^m} \frac{m^\star}{e_0} \right\}$$

$$\text{s.t.} \quad \frac{m^\star}{e_0} + \sum \frac{p(s)a^\star(s)}{e_0} \leq \overline{w}; \qquad m^\star \geq 0, a^\star(s) \geq 0 \; \forall s$$

- Money is weakly dominated $\Rightarrow \kappa(s) \geq \kappa^m$ for some s.
- Arbitrageurs invest in the highest return:

$$L = \overline{\kappa}\overline{w} \quad \text{where} \quad \overline{\kappa} \equiv \max_s \{\kappa(s)\} \geq \kappa^m$$

- Invest in money only if $\kappa(s) = \kappa^m$ for all s

Characterizing monetary equilibria: Implementability

Result (Implementation). An allocation $(c_0, \{c_1(s)\}, m)$ is part of an equilibrium if and only if

$$y_0 - c_0 - \sum_{s \in S} q(s) c_1(s) - \overline{\kappa w} = 0 \qquad \text{(IRC)}$$

$$h'\left(\frac{m}{e_0}\right) \frac{1}{u_0'(c_0)} = 1 - \underbrace{\sum_{s \in S} \frac{q(s) e_0}{(1+\kappa(s)) e_1(s)}}_{\frac{1}{1+i}} \qquad \text{(MD)}$$

$$\kappa(s) \geq 0 \; \forall s$$

$$\overline{\kappa} = \max_s \{\kappa(s)\}$$

Goal: Find the best allocation (in terms of HH's welfare)

The arbitrage return on money

Arbitrageur with one unit of the consumption good

- Invest it in domestic money:

  Cost today: 1 Benefit tomorrow: $\left\{ \frac{e_0}{e_1(s)} \right\}$

The arbitrage return on money

Arbitrageur with one unit of the consumption good

- Invest it in domestic money:
  Cost today: 1 Benefit tomorrow: $\left\{\frac{e_0}{e_1(s)}\right\}$

- Replicate that payoff abroad:
  Cost today: $\sum_s \frac{q(s)e_0}{e_1(s)}$ Benefit tomorrow: $\left\{\frac{e_0}{e_1(s)}\right\}$

The arbitrage return on money

Arbitrageur with one unit of the consumption good

- Invest it in domestic money:

  Cost today: 1 Benefit tomorrow: $\left\{ \frac{e_0}{e_1(s)} \right\}$

- Replicate that payoff abroad:

  Cost today: $\sum_s \frac{q(s)e_0}{e_1(s)}$ Benefit tomorrow: $\left\{ \frac{e_0}{e_1(s)} \right\}$

If $\sum_s \frac{q(s)e_0}{e_1(s)} > 1 \Rightarrow$

- Money (strictly) dominates foreign assets
- Return on money is a lower bound for the domestic bond

  $\Rightarrow$ Real rates of return cannot be equalized across regions

Optimal reserves: When ZLB is not binding

Case 1: $\sum_{s \in S} \frac{q(s)e_0}{e_1(s)} < 1$.

- In any equilibrium, ZLB is never binding.

Optimal reserves: When ZLB is not binding

Case 1: $\sum_{s \in S} \frac{q(s) e_0}{e_1(s)} < 1$.

- In any equilibrium, ZLB is never binding.

For case 1, the optimal allocation has $\kappa(s) = 0$ for all s.

- Possible implementation: $F(s) = 0$ for all s.
  - Backus-Kehoe (89) irrelevance of reserves (at the margin).

Optimal reserves: When ZLB binds

Case 2: $\sum_{s \in S} \frac{q(s) e_0}{e_1(s)} > 1$.

Optimal reserves: When ZLB binds

Case 2: $\sum_{s \in S} \frac{q(s)e_0}{e_1(s)} > 1$.

$\Rightarrow \kappa(s) > 0$ for some s.

Interlude 1: Reserves portfolio and arbitrage returns

- From the definition of $\kappa(s)$:

$$\kappa(s) = \frac{q(s)u_0'(c_0)}{\beta\pi(s)u_1'(c_1(s))} - 1$$

- How to increase $\kappa(s)$?
- CB accumulates assets that pay in state s
    - tilting consumption towards future
    - intervention large enough that private sector cannot undo it

The Central Bank's problem

$$V = \max_{(c_0, \{c_1(s)\}, m)} \left\{ u_0(c_0) + \beta \sum_s \pi(s) u_1(c_1(s)) + h\left(\frac{m}{e_0}\right) \right\}$$

subject to

$$y_0 - c_0 - \sum q(s) c_1(s) = \bar{\kappa} \bar{w} \qquad \text{(IRC)}$$

$$h'\left(\frac{m}{e_0}\right) \frac{1}{u_0'(c_0)} = 1 - \sum_s \frac{q(s) e_0}{(1+\kappa(s)) e_1(s)} \qquad \text{(MD)}$$

$$\bar{\kappa} = \max_s \{\kappa(s)\} \qquad (\bar{\kappa})$$

$$\kappa(s) = \frac{q(s) u_0'(c_0)}{\beta \pi(s) u_1'(c_1(s))} - 1 \geq 0 \ \forall s \qquad (\kappa(s))$$

The Central Bank's problem: A simplification

$$V = \max_{\bar{\kappa} \geq \kappa^m} \tilde{V}(\bar{\kappa})$$

where

$$\tilde{V}(\bar{\kappa}) \equiv \max_{(c_0, \{c_1(s)\})} \left\{ u_0(c_0) + \beta \sum_{s \in S} \pi(s) u_1(c_1(s)) + h(\bar{x}) \right\}$$

subject to

$$y_0 - c_0 - \sum_{s \in S} q(s) c_1(s) = \bar{\kappa} \bar{w} \qquad \text{(IRC)}$$

$$0 \leq u_0'(c_0) - \sum_{s \in S} \frac{\beta \pi(s) e_0}{e_1(s)} u_1'(c_1(s)) \qquad \text{(ZLB)}$$

$$0 \geq u_0'(c_0) - (1 + \bar{\kappa}) \frac{\beta \pi(s)}{q(s)} u_1'(c_1(s)) \ \forall s \qquad (\bar{\kappa})$$

Where are we going?

Two cases

Financially close

- $\overline{w} = 0$

Financially open

- $\overline{w}$ is large

Financially closed economy: $\overline{w} = 0$

$$V \equiv \max_{(c_0, \{c_1(s)\})} \left\{ u_0(c_0) + \beta \sum_{s \in S} \pi(s) u_1(c_1(s)) + h(\bar{x}) \right\}$$

subject to

$$y_0 - c_0 - \sum_{s \in S} q(s) c_1(s) = 0 \qquad \text{(IRC)}$$

$$0 \leq u_0'(c_0) - \sum_{s \in S} \frac{\beta \pi(s) e_0}{e_1(s)} u_1'(c_1(s)) \qquad \text{(ZLB)}$$

$$0 \geq u_0'(c_0) - (1 + \bar{\kappa}) \frac{\beta \pi(s)}{q(s)} u_1'(c_1(s)) \ \forall s$$

Financially closed economy: $\overline{w} = 0$

Result. $\exists \lambda > 0$ s.t.

- $1 + \kappa(s) = \dfrac{q(s) u_0'(c_0)}{\beta \pi(s) u_1'(c_1(s))} = \dfrac{1 - \lambda \frac{e_0}{e_1(s)} \frac{u_1''(c_1(s))}{u_1'(c_1(s))}}{1 + \lambda \frac{u_0''(c_0)}{u_0'(c_0)}} > 1$ for all s
- the nominal interest rate is zero.

Financially closed economy: $\overline{w} = 0$

Result. $\exists \lambda > 0$ s.t.

- $$1 + \kappa(s) = \frac{q(s) u_0'(c_0)}{\beta \pi(s) u_1'(c_1(s))} = \frac{1 - \lambda \frac{e_0}{e_1(s)} \frac{u_1''(c_1(s))}{u_1'(c_1(s))}}{1 + \lambda \frac{u_0''(c_0)}{u_0'(c_0)}} > 1 \text{ for all } s$$
- the nominal interest rate is zero.

Distort all returns but NOT necessarily equalize them.

Financially closed economy: $\overline{w} = 0$

Result. Suppose that (i) $\pi(s)/q(s)$ is constant and that (ii) u_1 is DARA. Then, $\kappa(s)$ is strictly decreasing in $e_1(s)$.

Financially closed economy: $\overline{w} = 0$

Result. Suppose that (i) $\pi(s)/q(s)$ is constant and that (ii) u_1 is DARA. Then, $\kappa(s)$ is strictly decreasing in $e_1(s)$.

$$c_1(s) = f(s) + F(s) - \frac{m^\star + a^\star(s)}{e_1(s)}$$

Financially closed economy: $\overline{w} = 0$

Result. Suppose that (i) $\pi(s)/q(s)$ is constant and that (ii) u_1 is DARA. Then, $\kappa(s)$ is strictly decreasing in $e_1(s)$.

$$c_1(s) = \qquad + F(s) - \frac{m^\star + a^\star(s)}{e_1(s)}$$

Financially closed economy: $\overline{w} = 0$

Result. Suppose that (i) $\pi(s)/q(s)$ is constant and that (ii) u_1 is DARA. Then, $\kappa(s)$ is strictly decreasing in $e_1(s)$.

$$c_1(s) = \qquad + F(s)$$

CB reserve policy is pinned down.

$F(s)$ is strictly decreasing in $e_1(s)$.

Result. CB holds assets that pay when currency appreciates.

Financially closed economy: $\overline{w} = 0$

Goal is to relax ZLB constraint. Fisher equation:

$$1 \geq \sum \left(\underbrace{\frac{\beta \pi(s) u_1'(c_1(s))}{u_0'(c_0)}}_{\frac{1}{1+r}} \times \underbrace{\frac{e_0}{e_1(s)}}_{\frac{1}{1+\pi}} \right)$$

Financially closed economy: $\overline{w} = 0$

Goal is to relax ZLB constraint. Fisher equation:

$$1 \geq \sum \left(\underbrace{\frac{\beta \pi(s) u_1'(c_1(s))}{u_0'(c_0)}}_{\frac{1}{1+r}} \times \underbrace{\frac{e_0}{e_1(s)}}_{\frac{1}{1+\pi}} \right)$$

Distort more states where appreciation is bigger

Financially open economy: $\overline{w}$ large

- Recall arbitrage losses: $L = \overline{\kappa}\overline{w} = \max_s\{\kappa(s)\}\overline{w}$
- Consider the following problem:

$$\min_{\{\kappa(s)\}} \left\{\max_s\{\kappa(s)\}\right\} \text{ subject to:}$$

$$0 \leq 1 - \sum_s \frac{q(s)e_0}{(1+\kappa(s))e_1(s)} \qquad \text{(ZLB)}$$

$$0 \leq \kappa(s) \quad \forall s$$

- Solution: $\kappa(s) = \kappa^m$ for all s
- Only allocation were foreigners willing to hold money

Financially open economy: $\overline{w}$ large

Assumption :

- $u_0(c_0) = c_0$ and that $u_1(c_1)$ is in DARA class

Financially open economy: $\overline{w}$ large

Assumption :

- $u_0(c_0) = c_0$ and that $u_1(c_1)$ is in DARA class

Result. For $\overline{w}$ large enough, the $\kappa(s) = \bar{\kappa}$ for all s is optimal

Proof $\exists W$ s.t for all $\bar{w} > W$, $\tilde{V}(\bar{\kappa}) \leq 0$ for all $\bar{\kappa} \geq \kappa^m$

Financially open economy: $\overline{w}$ large

- Optimal $F(s), A(s), M$ and T_0, $T_1(s)$?

Financially open economy: $\overline{w}$ large

- Optimal $F(s), A(s), M$ and T_0, $T_1(s)$?

- $M \geq e_0\bar{x}$ to guarantee enough money satiation
- Any budget feasible $A(s)$, T_0, $T_1(s)$

Financially open economy: $\overline{w}$ large

- Optimal $F(s), A(s), M$ and T_0, $T_1(s)$?

- $M \geq e_0\bar{x}$ to guarantee enough money satiation
- Any budget feasible $A(s)$, T_0, $T_1(s)$
- $F(s)$ needs to satisfy:

$$\sum_s q(s)F(s) = y_0 - c_0^e + \overline{w} > 0$$

$$F(s) \geq c_1^e(s)$$

Result. If $q(s)/\pi(s)$ is constant $\Rightarrow$ c_1^e constant $\Rightarrow$ invest in safe (real) assets.

Summary so far

- Under imperfect international arbitrage:
  - Reserve portfolio matters
  - Trade-off: ZLB vs resource losses
  - Optimality hinges on size of capital flows
  - Open – Invest in safe assets – make sure foreigners holds m
  - Closed – Invest in assets that pay when currency appreciates

Summary so far

- Under imperfect international arbitrage:
  - Reserve portfolio matters
  - Trade-off: ZLB vs resource losses
  - Optimality hinges on size of capital flows
  - Open – Invest in safe assets – make sure foreigners holds m
  - Closed – Invest in assets that pay when currency appreciates
- Next: how to measure the losses?
  - Rate of return differentials of assets in different currencies
  - Uncovered Interest Parity or Covered Interest Parity?

Uncovered interest parity

- From the definition of κ^m

$$1 + \kappa^m = \sum_{s \in S} \frac{q(s) e_0}{e_1(s)}$$

Uncovered interest parity

- From the definition of κ^m

$$1 + \kappa^m = \sum_{s \in S} \frac{q(s) e_0}{e_1(s)} = \sum_{s \in S} \pi(s) \frac{q(s)}{\pi(s)} \frac{e_0}{e_1(s)}$$

Uncovered interest parity

- From the definition of κ^m

$$1 + \kappa^m = \sum_{s \in S} \frac{q(s)e_0}{e_1(s)} = \sum_{s \in S} \pi(s) \frac{q(s)}{\pi(s)} \frac{e_0}{e_1(s)}$$

$$1 + \kappa^m = \mathbb{E}\left[\frac{q(s)}{\pi(s)}\right] \mathbb{E}\left[\frac{e_0}{e_1(s)}\right] + Cov\left[\frac{q(s)}{\pi(s)}, \frac{e_0}{e_1(s)}\right]$$

Uncovered interest parity

- From the definition of κ^m

$$1 + \kappa^m = \sum_{s \in S} \frac{q(s) e_0}{e_1(s)} = \sum_{s \in S} \pi(s) \frac{q(s)}{\pi(s)} \frac{e_0}{e_1(s)}$$

$$1 + \kappa^m = \mathbb{E}\left[\frac{q(s)}{\pi(s)}\right] \mathbb{E}\left[\frac{e_0}{e_1(s)}\right] + Cov\left[\frac{q(s)}{\pi(s)}, \frac{e_0}{e_1(s)}\right]$$

$$\kappa^m = \underbrace{\frac{1 + i(=0)}{1 + i^\star} \mathbb{E}\left[\frac{e_0}{e_1(s)}\right] - 1}_{\text{UIP gap}} + \underbrace{Cov\left[\frac{q(s)}{\pi(s)}, \frac{e_0}{e_1(s)}\right]}_{\text{risk premium}}$$

Uncovered interest parity

- From the definition of κ^m

$$1 + \kappa^m = \sum_{s \in S} \frac{q(s) e_0}{e_1(s)} = \sum_{s \in S} \pi(s) \frac{q(s)}{\pi(s)} \frac{e_0}{e_1(s)}$$

$$1 + \kappa^m = \mathbb{E}\left[\frac{q(s)}{\pi(s)}\right] \mathbb{E}\left[\frac{e_0}{e_1(s)}\right] + Cov\left[\frac{q(s)}{\pi(s)}, \frac{e_0}{e_1(s)}\right]$$

$$\kappa^m = \underbrace{\frac{1 + i(= 0)}{1 + i^\star} \mathbb{E}\left[\frac{e_0}{e_1(s)}\right] - 1}_{\text{UIP gap}} + \underbrace{Cov\left[\frac{q(s)}{\pi(s)}, \frac{e_0}{e_1(s)}\right]}_{\text{risk premium}}$$

- UIP gap not enough to identify κ^m

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s) \left[\frac{1}{e_1(s)} - \frac{1}{f} \right] = 0$$

where f is the forward exchange rate.

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s) \left[\frac{1}{e_1(s)} - \frac{1}{f} \right] = 0 \;\Rightarrow f = \frac{\sum q(s)}{\sum \frac{q(s)}{e_1(s)}}$$

where f is the forward exchange rate.

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s)\left[\frac{1}{e_1(s)} - \frac{1}{f}\right] = 0 \;\Rightarrow f = \frac{\sum q(s)}{\sum \frac{q(s)}{e_1(s)}}$$

where f is the forward exchange rate.

- Definition of κ^m:

$$1 + \kappa^m = \sum q(s) \frac{e_0}{e_1(s)}$$

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s) \left[\frac{1}{e_1(s)} - \frac{1}{f} \right] = 0 \;\Rightarrow f = \frac{\sum q(s)}{\sum \frac{q(s)}{e_1(s)}}$$

  where f is the forward exchange rate.

- Definition of κ^m:

$$1 + \kappa^m = \sum q(s) \frac{e_0}{e_1(s)} = \sum q(s) \times \frac{\sum \frac{q(s)}{e_1(s)}}{\sum q(s)} e_0$$

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s)\left[\frac{1}{e_1(s)} - \frac{1}{f}\right] = 0 \;\Rightarrow f = \frac{\sum q(s)}{\sum \frac{q(s)}{e_1(s)}}$$

  where f is the forward exchange rate.

- Definition of κ^m:

$$1 + \kappa^m = \sum q(s)\frac{e_0}{e_1(s)} = \sum q(s) \times \frac{\sum \frac{q(s)}{e_1(s)}}{\sum q(s)} e_0$$

$$\kappa^m = \underbrace{\frac{1 + i(= 0)}{1 + i^\star}\frac{e_0}{f} - 1}_{\text{CIP gap}}$$

Covered interest parity

- You have one unit of currency at $t = 1$. In IFM:

$$\sum q(s)\left[\frac{1}{e_1(s)} - \frac{1}{f}\right] = 0 \;\Rightarrow f = \frac{\sum q(s)}{\sum \frac{q(s)}{e_1(s)}}$$

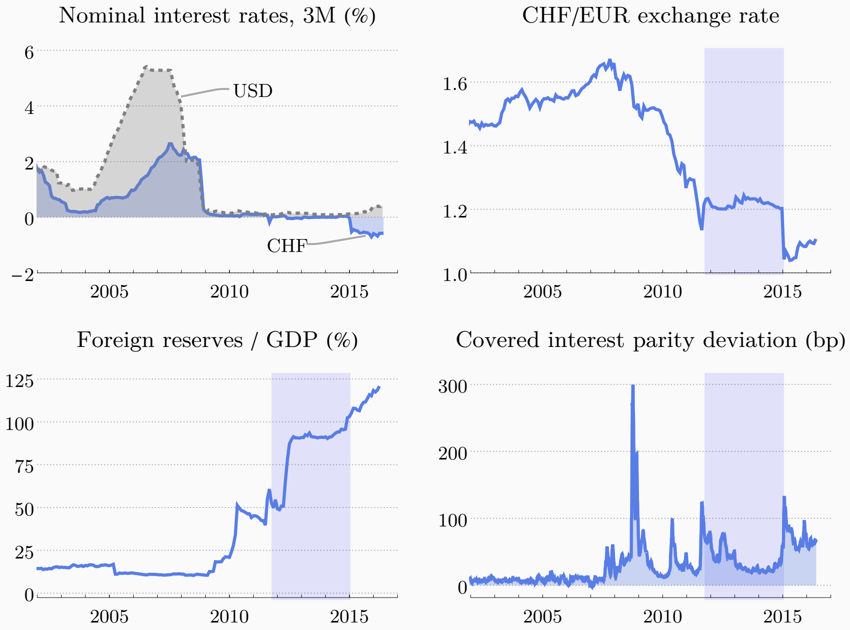
  where f is the forward exchange rate.

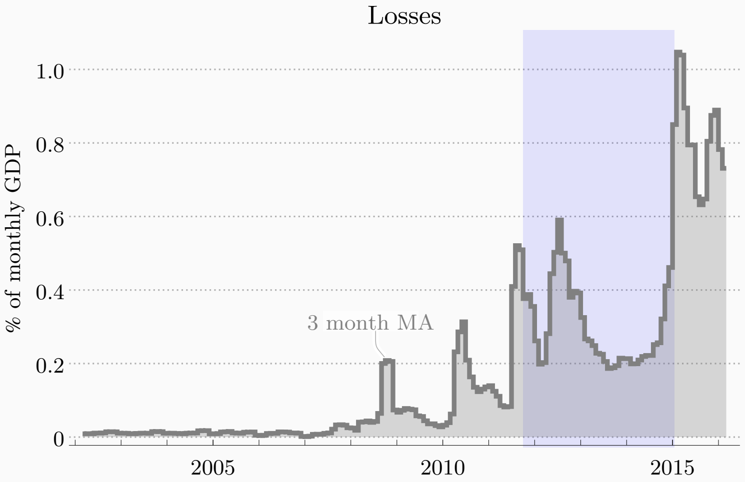
- Definition of κ^m:

$$1 + \kappa^m = \sum q(s)\frac{e_0}{e_1(s)} = \sum q(s) \times \frac{\sum \frac{q(s)}{e_1(s)}}{\sum q(s)} e_0$$

$$\kappa^m = \underbrace{\frac{1 + i(=0)}{1 + i^\star}\frac{e_0}{f} - 1}_{\text{CIP gap}}$$

- CIP gap equals κ_e (lower bound on losses)
- Large CIP deviations (Du, Tepper and Verdehaln, 2016)


Nominal interest rates, 3M (%)
6
4
2
0
−2
USD
CHF
2005
2010
2015
CHF/EUR exchange rate
1.6
1.4
1.2
1.0
2005
2010
2015
Foreign reserves / GDP (%)
125
100
75
50
25
0
2005
2010
2015
Covered interest parity deviation (bp)
300
200
100
0
2005
2010
2015


Losses
% of monthly GDP
1.0
0.8
0.6
0.4
0.2
0
3 month MA
2005
2010
2015

Arbitrageurs and domestic leverage

- Suppose now that $a^\star(s) \geq -e_1(s)\underline{a}^\star(s)$ for $\underline{a}^\star(s) > 0$.

$$\Pi = \overline{\kappa}\overline{w} + \sum_s (\overline{\kappa} - \kappa(s)) \frac{p(s)e_1(s)}{e_0} \underline{a}^\star(s)$$

$$= \overline{\kappa}\overline{w} + \sum_s q(s) \left(\frac{\overline{\kappa} - \kappa(s)}{1 + \kappa(s)} \right) \underline{a}^\star(s)$$

Arbitrageurs and domestic leverage

- Suppose now that $a^\star(s) \geq -e_1(s)\underline{a}^\star(s)$ for $\underline{a}^\star(s) > 0$.

$$
\begin{aligned}
\Pi &= \overline{\kappa}\overline{w} + \sum_s (\overline{\kappa} - \kappa(s)) \frac{p(s)e_1(s)}{e_0} \underline{a}^\star(s) \\
&= \overline{\kappa}\overline{w} + \sum_s q(s) \left(\frac{\overline{\kappa} - \kappa(s)}{1 + \kappa(s)} \right) \underline{a}^\star(s)
\end{aligned}
$$

- "Equal gaps" allocation, unchanged
  - if it was optimal before, remains optimal now

Arbitrageurs and domestic leverage

- Suppose now that $a^\star(s) \geq -e_1(s)\underline{a}^\star(s)$ for $\underline{a}^\star(s) > 0$.

$$\begin{aligned}\Pi &= \overline{\kappa}\overline{w} + \sum_s (\overline{\kappa} - \kappa(s)) \frac{p(s)e_1(s)}{e_0} \underline{a}^\star(s) \\ &= \overline{\kappa}\overline{w} + \sum_s q(s) \left(\frac{\overline{\kappa} - \kappa(s)}{1 + \kappa(s)} \right) \underline{a}^\star(s)\end{aligned}$$

- "Equal gaps" allocation, unchanged
    - if it was optimal before, remains optimal now
- Deviating from "equal gaps" creates additional losses
  even if $\overline{w} = 0$!.
- Role for policy

Conclusion

- Optimal porfolio hinges on *degree of openness*
  - Relatively closed economies:
    - Invest in foreign assets that pay when the currency appreciates
  - Relatively open economies:
    - Make sure that foreign investors demand domestic currency
    - Invest in safe assets
- CIP deviation – lower bound on potential losses
- Ability to leverage – makes thing worse for the CB
- Missing things:
  - No carry-trade – complete mkts (?)
  - Implications for a closed economy
  - Capital controls

Covered interest parity deviations (cross-section)