A Political Economy Model of Sovereign Debt Repayment*

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Abstract

Bulow and Rogoff (1989) show that a country that has access to a sufficiently rich asset market cannot commit to repay its debts and therefore should be unable to borrow. This is because for any debt contract, there exists a time at which the country is made better off by defaulting and replicating the payoffs of the debt contract through savings in the asset market. This paper provides an answer to this paradox based on a political economy model of debt. It shows that the presence of political uncertainty reduces the ability of a country to save, and hence to replicate the original debt contract after default. In a model where different parties alternate in power, an incumbent party with a low probability of remaining in power has a high short-term discount rate and is therefore unwilling to save. The current incumbent party realizes that in the future whoever achieves power will be impatient as well, making the accumulation of assets unsustainable. This time-inconsistency is shown to be equivalent to the problem faced by a hyperbolic consumer. Because of their inability to save, politicians demand debt ex-post and the desire to borrow again in the future enforces repayment today.

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Introduction

The history of sovereign lending is characterized by three broad facts: governments have at times been able to borrow substantial amounts of funds from foreign entities; much of what they borrowed they eventually repaid; and repayment was often complicated, involving delay, renegotiation, public intervention and default[1].

Sovereign debt is fundamentally different from private debt because a government cannot use many things as collateral for a loan and the ability to take a government to court is extremely limited. This gives rise to the question: Why do sovereign debtors pay back their debts?

The oldest explanation of why countries repay is that they must maintain a good reputation in foreign financial markets to be able to borrow more in the future. Eaton and Gertsovitz (1981) formalize this idea in the context of a small country subject to income shocks[2]. A defaulting government loses access to international credit markets. Default is costly because the country will not be able to smooth consumption later on. The desire to borrow in the future therefore induces the country to pay back its debts today.

This explanation is revisited by Bulow and Rogoff (1989). They show that if countries are able to save in rich asset markets, then reputation considerations alone cannot enforce repayment and countries will eventually default on any debt contract. The idea behind their argument is simple and illuminating: in any debt contract there is a point in time where the value of the debt of a borrower country reaches (or is very close to) a maximum. At that point the country would default and start a sequence of savings in a way that perfectly replicates the original debt contract but generates extra income (the interest that is not repaid). This sequence of savings is possible as long as the international markets offer a menu of assets indexed on the same contingencies as the original debt contract. So, if international asset markets are rich enough, countries will always default on their debts. Bulow and Rogoff conclude: "loans to LDCs are possible only if the creditors have either political rights, which enable them to threaten the debtor's interests outside its borrowing relationships, or legal rights".

Several other explanations of why countries repay their debts have been pro-

[1] Eaton and Fernandez (1995).

[2] Several authors have extended the reputation approach to sovereign lending. See for example Atkeson (1991), Grossman and van Huyck (1988), Worrall (1990).

posed. Researchers have studied the possibility that reputation spillovers to other valuable relationships might be costly enough to enforce repayment (Cole and Kehoe (1995), (1996)). Another approach looks at the assets available to the country after default. Technological restrictions (Kletzer and Wright (2000)) or collusion among banks (Wright (2002)) might reduce the range of savings mechanisms available to the country after default. Another branch of the literature studies the punishments available to creditors, from military intervention to trade embargoes[3].

This paper takes another look at reputation models of lending. I will argue that even when international financial markets are quite complete, political considerations restrain a country from implementing the saving sequence that the Bulow and Rogoff argument requires.

In order to arrive at this result the paper builds on the simple insight that politicians are not continuously in power. Because the nature of the political process does not assure the incumbent politician that he will be in power again tomorrow the politician is impatient. This impatience has already been used to explain politicians reluctance to save (Alesina and Tabellini (1990)). Incumbent politicians have a bias towards the present but this bias does not affect their discount rates between dates in the distant future. They are more patient in the distant future than they are today because the uncertainty over who will be in power in the future has yet to be revealed. They know however, that when tomorrow arrives, whoever is in power will be impatient in the short-run as well. This time-inconsistency can generate strong inefficiencies in the savings done by governments. I argue that it can also explain two seemingly contradictory facts:

- Politicians don't save and they spend too much.
- Most of the time, they pay back their debts even in the absence of punishments or clear political costs.

This paper shows that political uncertainty generates inefficient savings and makes the replication strategies of Bulow and Rogoff (1989) not possible. A political economy model with political turnover is presented and the equilibrium behaviour of the

[3] In a very interesting paper, Rose (2002) has shown that after a country defaults, its international trade is significantly reduced, identifying a channel through which external creditors might be punishing the defaulting country.

parties is characterized. It is shown that when political turnover is positive, parties tend to consume too much out of the stock of assets of the country.

In the Bulow and Rogoff (1989) argument the country is always better off by defaulting on any debt contract. The country can save in the asset markets and generate the same consumption allocation that the debt contract was generating (without having to pay back the interest rate). However, the presence of political uncertainty reduces the ability of the country to keep the assets around for long. The parties in power realize that if they were to default, future governments will inefficiently overspend and the country will run out of its assets too fast. Because they might be in power again in the future, this inefficient overspending lowers their utility today.

Debt reduces this inefficiency. The reason is that the parties in power can borrow from foreigners when the asset stock is low enough. This in turn implies that by repaying their debts, they do not have to keep a high stock of assets around for smoothing purposes (they can borrow when times are bad), and this reduces the temptation of future governments to inefficiently overspend from accumulated savings. This improvement in the allocation of resources can be valuable enough for the parties today to enforce repayment of previous debts.

The paper relates to the political economy literature on fiscal deficits (Alesina and Tabellini (1990), Persson and Svensson (1989)). However, these papers do not consider the possibility of default. Tabellini (1991), and Dixit and Londregan (2000) present models of sustainability of domestic debt[4]. In these models, the lenders are citizens and thus have political rights (they can vote). I analyze a model of *sovereign* debt, where lenders reside outside the country and have no political rights.

The paper builds on the techniques developed by Harris and Laibson (2001) in their characterization of the hyperbolic consumer problem. These techniques proved very useful in the analysis of the political game.

Finally, the paper is related to recent work by Gul and Pesendorfer (2002). In their work, the authors develop a theory of of preferences for commitment, providing a modeling alternative to the standard hyperbolic discounting framework. They study a consumer with these preferences and show how Bulow and Rogoff (1989)'s result might be overturned. Their model does not connect to political economy, as this

[4]For other papers in intergenerational redistribution see Rotemberg (1990), Grossman and Helpman (1998) and Mulligan and Sala-i-Martin (1999).

paper does; and hence does not have clear empirical predictions for sovereign debt. The main motivation behind their work is the desire to eliminate the multiplicity of "selfs" that appears in the hyperbolic discounting literature. While this is a desirable characteristic for an intra-personal game; in a political game the multiplicity of decision makers seems to be a much better approximation of reality.

The sequence of the paper is as follows. First, I setup the model without debt. The model consists of a small economy with different political parties subject to endowment and political shocks. I define the equilibrium and characterize some of its main properties. This is done in Section 2. Section 3 analyzes the model at the limit when the political shocks are very likely. For this case I have closed form solutions for the equilibrium and a uniqueness result. Section 4 introduces the possibility of borrowing from outsiders and characterizes the equilibrium behaviour of the parties when they have access to international lending. Section 5 presents the argument of debt sustainability. I characterize under which conditions the country repays its debts and under which it will default. I show that the argument of Bulow-Rogoff does not in general hold except in the particular case when there is no political turnover. Section 6 shows the differences in debt sustainability that should be expected across political systems. Section 7 concludes. Before entering into the model, I quickly review some of the relevant empirical literature.

1 Empirical Evidence

There were several historical instances where the accumulation of state surpluses was politically impossible. Cole, Dow and English (1994) report the interesting case of the United States in the mid-1830s. At that time, the accumulation of a large federal surplus was controversial and at the end, the surplus was distributed to the states. The states did not hold the money for long, and spend or distributed it. Years later, Benjamin R. Curtis (a supreme court judge) specifically argued that a state's reputation in credit markets was important because U.S. states could not accumulate surpluses, and in an emergency they might need more resources than they could tax in a single year. Alexander Hamilton made a similar point in the case for repayment of the U.S. Revolutionary War debt.

In their analysis of international reserve-holding behavior of developing countries, Aizenman and Marion (2002) provide evidence that countries with higher political uncertainty (measured as the probability of a leadership change) tend to accumulate lower levels of reserves. Their argument is that higher political uncertainty reduces the optimal size of buffer stocks held by a government because it increases the opportunistic behavior of the policy maker.

Political uncertainty has been associated to other fiscal problems. For example, Cukierman, Edwards and Tabellini (1992) document the fact that higher political uncertainty is positively associated with seignorage. They argued that seignorage reflects the high costs of administering and enforcing the collection of regular taxes, but that the evolution of the tax structure of a country depends on the political system. When there is high political turnover, incumbent politicians might choose to maintain an inefficient tax system so as to constrain the behavior of future governments, which current incumbents might disagree with.

Political uncertainty tends to be associated with inefficient fiscal behavior in general. Governments seem to have a lower ability to save, a more inefficient tax system and more problems controlling spending, the higher the political uncertainty faced by incumbents is. In this paper I argue that these inefficiencies[5], in particular the savings one, might be the reason why governments repay their foreign debts even in the absence of punishments or direct political costs.

2 The Model

In this section the political model is set up. I first analyze the equilibrium behavior of the political parties without debt. In later sections the possibility of sovereign lending is introduced.

Consider a small economy which has m political parties indexed by i. Each party has the following utility defined over the continuous flow of consumption provided by the government at every instant t.

$$U_0^i = E_0 \left[\int_0^\infty e^{-rt} u\left(c_t^i\right) dt \right]$$

where c_t^i is the consumption provided to party i at time t by the government.

[5] Lane (2000) does a detailed analysis of international lending to LDCs and finds evidence that better government anti-diversion policies (policies that reduce rent-seeking activities by politicians) are associated with lower amounts of sovereign borrowing. This negative relationship weakens as other controls are included, but surprisingly enough it never becomes positive.

Assume the CRRA utility representation

$$u(c) = c^{1-\rho}$$

with[6] $0 < \rho < 1$.

For notation purposes, all stock variables are uppercase while flow variables are lowercase.

Every instant the party in power decides how much to provide to different parties and how much to save in an asset market.

How does the government finance its spending flows?

• With Poisson probability λ there is an endowment shock.

• Immediately after the shock, the country receives a stock Y of income.

• The rest of the time, the country does not receive any endowment and spends out of previous savings.

At any point in time a given party control the government. I proceed now to characterize how power is allocated among the different parties. The following simple political structure is assumed[7]:

• With Poisson probability γ there is a political shock.

• Immediately after the shock, a party is randomly chosen to govern.

• The probability that a party is selected[8] is $\alpha \in (0, 1)$.

The political shock and the endowment shock are assumed to be independent events.

Let p_t be the party in power at time t.

Notice that the parties consume only through government provision. This is the case if the government is the only entity that can provide the public goods that the parties desire (like roads, schools, dams, etc.).

Where do these political shocks come from? They could be the outcome of elections, strikes that force a change in government, variability in the bargaining power

[6] Under this condition ($0 < \rho < 1$) we have that $u(0) = 0$. This is important because parties do not always consume in the equilibrium, and utility comparisons cannot be made unless $u(0) > -\infty$.

[7] In a previous version of the paper we had a more general political structure. Ruling coalitions were selected out of the political parties, and these coalitions decided on spending and savings. A ruling coalition was constrained to provide the same consumption to the parties in the coalition. This structure generalized the set up but did not add anything to the results and made the notation cumbersome.

[8] We will think of α as continuous from (0,1). One can reinterpret this political set up as a coalitional set up where α could take any value. See previous footnote.

of the different parties in the coalition, the possibility of impeachments, or just the political breakdown of the ruling coalition. The current incumbents always face this risk and this uncertainty make them impatient about the future. The incumbents do not know for sure that they will be governing again tomorrow.

Assumption 1 *There is* ***political turnover****:* $\gamma(1-\alpha) > 0$ *and* $\alpha > 0$*.*

From standard concave utility arguments the parties have a desire to smooth through time the consumption flow, and the ruling parties would like to save part of the government's income for the future. However, there is a chance that current incumbents won't be in power tomorrow to consume out of the savings they made today. This reduces the amount they save. Savings are shown to be inefficient from the perspective of all parties. Politicians consume too much out of their stock of assets.

The options available to the government for savings are specified in the next section.

2.1 (Cash-in-Advance) Asset Market

There is a foreign spot asset market populated by foreign investors that are risk neutral and share the same discount rate r. The government can save in the foreign spot asset market.

There is a riskless bond that returns a constant flow of r (note that parties and the outsiders share the **same discount factor**). Let B_t denote the holdings on the bond a country has at any time t.

The other relevant asset is a Lucas tree. The return on this tree is assumed to be contingent on the realization of the country's endowment shock. The reason why this asset is introduced is the following[9]. The Bulow-Rogoff argument says that if a country has access to an asset market that provides contracts (assets) indexed by the same contingencies as a debt contract, then the country will at some point in time default on this debt contract and will start saving in the asset market. The main point of this paper is to show how the ability of the country to sustain debt changes when political risk is introduced into a model where the Bulow-Rogoff result would

[9] To the reader familiar with the Bulow and Rogoff (1989) argument, this Lucas tree represents the cash in advance contracts available to a country upon default.

otherwise hold (something that is expected to happen with the introduction of this contingent asset).

Let A_t be the holdings of this tree at time t and A_{t+dt} be the holdings of the tree at time $t + dt$. Then

$$A_{t+dt} = \begin{cases} 0 & \text{; if the endowment shock happens} \\ (1 + (r + \lambda)\, dt)\, A_t & \text{; if the endowment shock doesn't happen} \end{cases}$$

The return from holding the tree is $r + \lambda$ when there is no endowment shock[10].

This contingent Lucas tree could be thought of as the result of the ability of foreign investors to make instantaneous commitments contingent on the aggregate endowment shock. In any interval Δt, the foreign investors can write a short term contract (that lasts only one Δt period) contingent on the realization of the endowment shock. Competition on the investors' side will tie the return on the asset to the zero profit condition. As Δt goes to zero the instantaneous Lucas tree is obtained.

The assumption underlying this contingent Lucas tree is that foreigners have a commitment technology that allows them to credibly offer these instantaneous cash-in-advance contracts to the country. These are contracts where the country pays upfront and receives non-negative payments ex-post. The case of interest is when the country *cannot* commit to repay its past obligations with the foreigners and can default on a contract that requires it to make a payment ex-post. I first study the case when the government cannot borrow from foreign investors and I analyze the ability to repay in later sections.

Let W_t denote the total asset income of the country. The following is assumed

Assumption 2 *The country cannot short the riskless bond :* $A_t \leq W_t$

$B_t = W_t - A_t$ represents the investments done in the riskfree bond at time t. Then, the amount that is invested in the riskless bond, B_t, cannot be negative. This assumption will be relaxed in section 4 once the possibility of borrowing from abroad is introduced.

[10] Why is the return $r + \lambda$? Let W be the value of holding T units of the contingent tree for a risk neutral investor. W is given by the value equation $rW = \hat{r}T + \lambda(-T)$; where $\hat{r}$ is the return on the tree. Given that the asset is unit priced and a zero profit condition holds we have that $W = T$ (the value of holding the T units of the tree for a foreign investor is equal to its price). From this we obtain that $\hat{r} = r + \lambda$.

Summing up, the country is subject to two shocks: endowment and political shocks. The endowment shocks create a desire for smoothing the government provision flow. In the next section I characterize how the political forces interact with the smoothing needs of the parties.

2.2 The Political Equilibrium

Let a provision profile $\hat{c}_t$ be the vector of spending allocations to every party at time t, $\hat{c}_t = (c_t^1, c_t^2, ..., c_t^m)$. Any point in time is characterized then by the vector $h_t = \{p_t, \hat{c}_t, A_t\}$ detailing the ruling party, the spending allocation done and the savings (portfolio) decision A_t. A history is then a correspondence from the $[0, T)$ to possible vectors h_t.

A strategy for party i at time t is a mapping from all possible histories up to time t and states at time t where party i is in power to instantaneous spending allocations and savings decisions.

The nature of this dynamic game allows for multiple subgame perfect equilibria. These equilibria are difficult to characterize, and in a related paper (Amador (2002a)) I study the asset management when the parties are constrained to play only equilibria in the pareto frontier of the perfect payoff set. In this paper, however, I will use a different characterization.

Definition 1 *A **Stationary Markov strategy** for party i is a profile of consumption $\hat{c}_i(W) = (c_i^1(W), c_i^2(W), ..., c_i^m(W))$ and an investment function $A_i(W)$.*

The correspondence $\hat{c}_i(W)$ determines the consumption allocation to all parties that party i will choose if she is in power at some time with an asset level of W. A consumption allocation $c_i^j(W)$ is the consumption party i will provide to party j if she were in power with W assets. The function $A_i(W)$ details how much of the savings are done in the contingent Lucas tree if party i is in power with W assets.

The following is assumed

Assumption 3 *Parties play only stationary Markov strategies.*

So, politicians only play strategies that are a function of the payoff relevant variables at a given point in time. There is no reputation building under this assumption. Whatever happened in the past that does not affects the income of today and the future does not matter for the politicians' behaviour. The paper focuses on Markov Perfect Equilibria, i.e. subgame perfect equilibria that use Markov strategies.

Suppose that party i is the ruling party. Let t_1 be the time when the first political shock arrives after $t = 0$. Then, the utility of party i today is:

$$V(W_0) = E_{t_0}\Big[\int_0^{t_1} e^{-rt}u\left(c_i^i(W(t))\right)dt + e^{-rt_1}\left[\alpha V(W(t_1)) + (1-\alpha)V_o(W(t_1))\right]\Big]$$

where V is the expected utility of the incumbent at time 0 with asset level W_0 and V_o is the expected utility of the party once out of power. This equation tell us that the party in power consumes up to the time when the political shock arrives (t_1). At that time, with probability α the party in power remains as the incumbent and receives a value of V; and with probability $(1-\alpha)$ she is out of power and receives a value V_0. Below, I will be more specific about the latter value function.

The party i faces (in the absence of an endowment shock) the following instantaneous budget constraint

$$dW = \left((r(W - A_i) + (r+\lambda)A_i) - c_i^i - \sum_{j\neq i} c_i^j\right)dt$$

and if $W = 0$, then $c_i^j = 0$ for all j.

To understand this equation, notice that $r(W - A_i) + (r+\lambda)A_i$ is the return on the asset holdings if A_i is the amount of wealth invested in the contingent Lucas tree: the return on the riskless bond is $r(W - A)$ and the return on the contingent tree is $(r+\lambda)A$. And $c_i^i(W) + \sum_{j\neq i} c_i^j(W)$ is the total spending flow done by the government. The change in wealth (dW) is then the return from holding the assets minus the spending done in that instant. When there is an endowment shock , the asset level W_t will jump to $(Y + W - A)$.

Once a political shock arrives, only total wealth at that time matters. Sharing of the spending in the past is irrelevant for future play. So, the party in power today maximizes $V(W)$ and for a given total amount of spending spends everything on its own consumption. There is no reason to share with the outsiders if tomorrow's play will not be affected by it. The following proposition states this.

Proposition 1 *In a Markov Equilibrium, for any $i \neq j$, $c_i^j = 0$ for almost all W.*

This reduces the level of complexity of the problem considerably. The party in power always gives zero provision to the outsiders. The only decision left to be made at time t is the amount of total spending c_t, subject to the budget constraint. The symmetry of the game implies that any party in power at time t faces exactly the same problem. I concentrate then on symmetric equilibria.

Definition 2 *A control $x = (c, A)$ is* ***feasible*** *if c and A are such*

- $c : [0, \infty) \to [0, \infty)$ *with* $c(0) = 0$
- $A : [0, \infty) \to (-\infty, \infty)$ *with* $A(W) \leq W$

This definition tell us that a control is feasible if it satisfies the short-sale constraint ($A(W) \leq W$) and the parties cannot consume when there is no wealth ($c(0) = 0$).

Given a control x the evolution of wealth is defined as follows.

Definition 3 *For a given control $x = (c, A)$,* ***wealth evolves according to*** x ***from*** W_0 *if when the endowment shock does not occur the asset level of the country follows*

$$dW = (rW + \lambda A(W) - c(W))\, dt \tag{1}$$

where $W(t)$ jumps to $(Y + W(t) - A(W(t)))$ when the endowment shock occurs; and $W(0) = W_0$.

For a given control (c, A), every instant the country receives a flow of income equal to $rW + \lambda A$; where rW is the return from all the asset holdings, and λA is the extra return from the holdings of the Lucas tree. This flow of income minus the consumption flow is equal to the instantaneous change in wealth.

Let W_y^x denote the case when W is evolving according to x from y .

For two feasible controls $x = (c, A)$ and $x_1 = (c_1, A_1)$ the value $V(W|x_1, x)$ is defined to be the expected value for an incumbent party if she follows (c_1, A_1) and everybody else follows (c, A)

$$V(W_0|x_1, x) = E\left[\int_0^{t_1} e^{-rt} u\left(c_1\left(W_{W_0}^{x_1}(t)\right)\right) dt + \right.$$
$$\left. + e^{-rt_1}\left(\alpha V\left(W_{W_0}^{x_1}(t_1)|x_1, x\right) + (1-\alpha) V_o\left(W_{W_0}^{x_1}(t_1)|x_1, x\right)\right)\right]$$

where

$$V_o(W_0|x_1, x) = E\left[e^{-rt_1}\left[\alpha V\left(W_{W_0}^{x}(t_1)|x_1, x\right) + (1-\alpha) V_o\left(W_{W_0}^{x}(t_1)|x_1, x\right)\right]\right]$$

and where t_1 is the first time after $t = 0$ when a political shock arrives.

The value function $V(\cdot|x_1, x)$ captures the value for an incumbent of following the strategy x_1 when everybody else follows x. The incumbent consumes c_1 as long as she is in power and wealth is evolving according to her strategy (x_1). When a political shock arrives (at t_1), with probability α the current incumbent remains in power, and with probability $(1-\alpha)$, she is out of office. Once out of office, she receives a utility level captured by the value function $V_o(\cdot|x_1, x)$. This value function tells us that the party out of power does not consume while out of government and wealth is evolving according to the strategies of the other parties (x). Once a political shock arrives, with probability α the party becomes the incumbent, and with probability $1-\alpha$, she remains an outsider.

A Symmetric Markov Equilibrium is defined as follows:

Definition 4 *A **Symmetric Markov Equilibrium** is a feasible control* $x^* = (c^*, A^*)$ *such that for any other feasible control* $x = (c, A)$*,*

$$V(W|x^*, x^*) \geq V(W|x, x^*) \qquad ;\text{for } W \in [0, \infty) \tag{2}$$

for all $W \in [0, \infty)$

The definition tells us that a symmetric Markov equilibrium is characterized by two functions c^* and A^* such that for any other feasible functions c and A the value generated by following the first strategies is at least as high as the value generated by following c and A while the given party is in power and when everybody else follows c^* and A^*.

Notice that a symmetric Markov equilibrium as defined above is subgame perfect. It is not difficult to show that a best response to a Markov strategy is also Markov. So, equation (2) is enough for perfection.

In general there may be many solutions to (2). The method used to select among equilibria is the technique developed by Harris and Laibson (2001) in their study of a hyperbolic consumer problem. The technique is described in detail in the Appendix I. The reason why this technique can be used is that a solution to (2) is equivalent to a Markov solution of a well-chosen hyperbolic program.

2.2.1 The Hyperbolic Equivalence

To show this result, notice that $\exists W \in [0,\infty)$ such that $V(W|x_1,x) > V(W|x,x)$ if and only if

$$E\left[\int_0^{t_1} e^{-rt} u\left(c_1\left(W_{W_0}^{x_1}(t)\right)\right) dt + \right.$$
$$\left. + e^{-rt_1}\left(\alpha V\left(W_{W_0}^{x_1}(t_1)|x,x\right) + (1-\alpha) V_o\left(W_{W_0}^{x_1}(t_1)|x,x\right)\right)\right] > V(W_0|x,x) \quad (3)$$

for some $W_0 \in [0,\infty)$, where the main difference between (3) and (2) is that (3) considers only deviations by incumbents during the period before the *first* political shock arrives.

Now, let

$$J(W|x) = \frac{1}{\alpha}\left[\alpha V(W|x,x) + (1-\alpha) V_0(W|x,x)\right]$$

Using the value functions and solving out,

$$J(W_0|x) = E\left[\int_0^{\infty} e^{-rt} u(c(W_{W_0}^{x}(t)))\right]$$

So, $J(W|x)$ is the utility of a party that uses a control x and is continuously in power.

Let the value function $\tilde{V}$ be defined as

$$\tilde{V}\left(W_0|x_1,x\right) = E\left[\int_0^{t_1} e^{-rt} u\left(c_1\left(W_{W_0}^{x_1}(t)\right)\right) dt + e^{-rt_1}\left(\alpha J\left(W_{W_0}^{x_1}(t_1)|x\right)\right)\right]$$

Notice that $\tilde{V}$ corresponds to the left hand side of (3). Then, x^* is a Symmetric Markov Equilibrium if for any feasible control x,

$$V\left(W|x^*,x^*\right) \geq \tilde{V}\left(W|x,x^*\right)$$

for $W \in [0,\infty)$

The value function[11] $\tilde{V}$ is equivalent to the value function of a hyperbolic consumer which faces a vanishing "present" (from 0 to t_1), and discounts all the "future" (from t_1 onwards) by $\alpha < 1$. The techniques of Harris and Laibson (2001) can then be used to characterize the political equilibrium. For expositional purposes, the main body

[11] Notice that $\tilde{V}(W|x,x) = V(W|x,x)$

of the paper presents only the results and refrains from developing the techniques in detail (this is done in the appendix).

2.2.2 The Bellman System

The following proposition states the associated Bellman equation for a Markov equilibrium[12].

Proposition 2 *A Symmetric Markov Equilibrium is a feasible control* $x^* = (c^*, A^*)$ *such that* $\exists V, V_0$ *where* (c^*, A^*) *solve*

$$rV(W) = \max_{c,A} \; u(c) + \lambda\left[V(Y+W-A) - V(W)\right] + V'(W)(rW + \lambda A - c) + \gamma(1-\alpha)(V_o(W) - V(W)) \quad (4)$$

where V_o *is given by*

$$rV_o(W) = \lambda\left[V_o(Y+W-A) - V_o(W)\right] + V_o'(W)(rW+\lambda A - c) + \gamma\alpha(V(W) - V_o(W)) \quad (5)$$

The two value functions V and V_o capture the expected utility for a party in or out of power respectively. The Bellman equations have a very intuitive expected utility interpretation:

The first equation tell us that the current utility flow for a party in power is equal to the consumption flow she receives plus the probability that the endowment shock arrives (λ) times the corresponding change in the value function ($V(Y+W-A) - V(W)$), plus the change in value due to accumulation or decumulation of the asset stock ($V'(W)(rW+\lambda A-c)$) and plus the probability that a political shock arrives and the current incumbent is not in power anymore ($\gamma(1-\alpha)$) times the corresponding change in value ($V_o(W) - V(W)$).

The second equation tell us that the current utility flow of being out of power is equal to the probability that an endowment shock arrives (λ) and the asset level moves from W to $Y+W-A$ times the corresponding change in value $[V_o(Y+W-A) - V_o(W)]$, plus the change in value due to accumulation or decumulation of the asset stock

[12] See Appendix for a formal derivation.

$\left(V_o'(W)(rW+\quad \lambda A-c)\right)$ and plus the probability that a political shock arrives and the party is in power $(\gamma\alpha)$ times the corresponding change in value $(V(W)-V_o(W))$.

The main difference between being in power and not is two fold :

• The parties not in power do not receive a government provision.

• The parties not in power have no decision to make, while the party in power selects the spending flow for the instant.

Taking the first order condition of the system with respect to c, when $W>0$, the following equation is obtained:

$$u'(c^*)=V'(W)$$

The current flow of spending is only constrained when $W=0$. For any $W>0$, consumption is unconstrained and the first order condition will hold with equality. This first order condition is also sufficient for optimality, because W is fixed at any instant and u is concave by assumption. The condition says that the marginal utility of consumption is equal to the marginal value of wealth. Differentiating this equation with respect to the state variable

$$u''(c^*(W))\,c^{*\prime}(W)=V''(W)$$

By concavity of u, we have that $u''<0$. As long as spending is monotonically increasing in W, $c^{*\prime}(W)>0$, the value function is also concave in W.

Taking the first order condition with respect to A (for $W>0$):

$$V'(Y+W-A)\leq V'(W)$$

with equality for $A<W$.

Suppose now that $W\leq Y$. In this case for any $A<W$, $Y+W-A>W$. If the value function is strictly concave (if $c^{*\prime}(W)>0$) then $V'(Y+W-A)<V'(W)$. This does not satisfy the first order condition (which holds with equality for $A<W$), so A has to be W: when $W\leq Y$, then $A^*=W$, i.e. all the savings are done in the contingent tree.

When $W>Y$, then from the first order condition $A^*=Y$. The following result obtains

Result 1 *If $c^{*\prime}(W) > 0$ for all $W > 0$, then the following holds in equilibrium*

$$A^*(W) = \begin{cases} W & \text{for all } W \leq Y \\ Y & \text{for all } W > Y \end{cases} \tag{6}$$

Notice that A^* does not depend on α or γ. The instantaneous portfolio decision is not affected by the political distortions. The political distortions affect the aggregate level of consumption and eventually whether or not W is less than Y, but not the way assets are allocated at any instant for *a given* W. This means that there are inefficiencies not because the incumbent is not doing the savings *right* in the sense that it is not using the asset market efficiently but rather that it will be consuming too much. This will become clear when I analyze the efficiency of the political equilibrium, which is done in the next section.

2.3 Constrained Efficiency

In this section I study the constrained efficient solution to the savings problem.

If there were no political turnover, $(\gamma(1-\alpha) = 0)$,the incumbent party maximizes a standard exponential problem. However, when there is political turnover $(\gamma > \gamma(1-\alpha) > 0)$ a Markov equilibrium is clearly inefficient. Parties that are not in power do not receive any provision allocation from the government. I call this inefficiency the "sharing" inefficiency. There is however another inefficiency. The perceived return on the savings by the party in power today has been reduced because of political risk. Incumbents save too little. I call this the "savings" inefficiency.

Why does the savings inefficiency happen? Suppose that the country has just received a political shock but the uncertainty about the ruling party has not yet been realized. From the perspective of all parties, they all have the same probability (α) of being in power, so they are identical. I ask the question: assuming that only the political winners receive a government provision, if the parties could commit to a provision rate (without knowing who among them will be in power) what rate would they pick? The parties maximize their ex-ante value (before knowing who will be in power).

Recall that $J = \frac{1}{\alpha}[\alpha V + (1-\alpha)V_o]$. The ex-ante value (under committment) is then given by $\alpha J^C = \alpha V^C + (1-\alpha)V_o^C$. The optimal commitment solution $\left(c^C, A^C\right)$

is such that

$$rJ^C(W) = \max_{c,A} u(c) + \lambda \left[J^C(Y + W - A) - J^C(W) \right] + J^{C\prime}(W)(rW + \lambda A - c) \quad (7)$$

This program is a standard exponential program. The consumption rate that all parties would like to commit to before the uncertainty about the political shock is realized is exactly the same as the one that a party continuously in power would pick. This is a constrained efficient result: it is the rate that a central planner constrained to provide consumption flows only to parties in power would pick. The following holds

$$c^C(W) = \begin{cases} (r+\lambda) W & ; \text{for } W \in (0, Y] \\ rW + \lambda Y & ; \text{for } W > Y \end{cases} \quad (8)$$

The intuition for this result is very simple. Because the interest rate of the assets and the discount rate of the parties are the same, the optimal spending is to mantain the level of wealth constant across time and states of nature. Given that there is a borrowing constraint, this desire implies that the consumption flow is $(r+\lambda) W$ whenever wealth is below Y (where $(r+\lambda) W$ is the return from holding all the assets in the contingent tree). And the consumption flow is $rW + \lambda Y$ when wealth is above Y, where this is the return from holding the assets in this case (Y is now the amount invested in the contingent tree). The constrained efficient solution is then to mantain the asset level. Notice that this will also be the aggregate spending that an unconstrained social planner will pick (a social planner that provides consumption to all parties, irrespective of whether they are in power or not).

Political uncertainty distorts the savings decision. Once the uncertainty about the political shock is realized, the incumbents choose (as it is shown in the next sections) a provision flow equal to $c^*(W) > c^C(W)$. There is too much spending from the ex-ante perspective of all parties. Notice that the distortion of savings (savings inefficiency) is the outcome of the inability of the parties to share the intra-instant provision flows (sharing inefficiency), but it is different in nature. For example, this savings inefficiency can clearly be reduced if the government has access to an illiquid savings technology. Even if the illiquid technology could do nothing to improve the sharing within a given instant it would constrain the parties to an aggregate consumption flow that is smaller than the one they would otherwise choose. However, in the current set-up, all assets available to the government are assumed to be completely liquid.

2.4 A Description of the Equilibrium

Suppose for now that the value function of being in power is concave. For the particular case of $W \leq Y$, the following results hold

Proposition 3 *For any* $\alpha < 1$*,*

$$c^*(W) > c^C(W) = (r + \lambda) W$$

For $\alpha < 1$, politicians are consuming faster out of the asset stock than a central planner would. This means that starting from $W \leq Y$, the wealth process never leaves the interval $[0, Y]$.

Politicians also consume faster the higher is the political uncertainty.

Proposition 4 *For any* $\alpha < 1$*,*

$$\frac{dc^*(W)}{d\gamma} > 0$$
$$\frac{dc^*(W)}{d\alpha} < 0$$

The higher is the probability of a political shock (γ) the faster the incumbent will run out of asset holdings.

The higher is the political risk (lower α), the faster the incumbent will run out of asset holdings.

This description of the equilibrium will be valid as long as the value function V is concave for all the domain of W ($W \in [0, \infty)$). However, in general there might be cases where $c'(W) < 0$ and hence V will be convex for certain values of W. This implies that the optimal portfolio decision A isn't in general continuous or monotonic in W. To be able to generalize the intuitions of this section, the techniques developed by Harris and Laibson (2001) in their study of the hyperbolic consumer when the "present" vanishes away are used. In my case, this implies the study of the economy as the political shock becomes very likely ($\gamma \to \infty$).

3 The Limit Economy

In this section the equilibrium as $\gamma \to \infty$, or when there is a political shock every instant is studied. See Appendix I for a full derivation of the setup. I call the limit policy functions the policy functions of the limit economy.

This rest of the paper exploits the continuous time setup. It is possible to obtain closed form solutions for the equilibrium and we can compute comparative statics.

There are two cases to consider.

3.1 Case 1: $\alpha + \rho - 1 > 0$

Suppose that $\alpha+\rho-1 > 0$. This assumption is satisfied when the political uncertainty and the elasticity of substitution are sufficiently small.

The first fundamental result is

Proposition 5 *In the limit economy, the value function of being in power is concave.*

This proposition tell us that equation (6) characterizes the optimal investment strategy.

The following proposition is obtained.

Proposition 6 *In the limit economy, the value function of being in power is, for* $W \leq Y$ *:*

$$rV_{\infty}(W) = \frac{r}{r+\lambda}\psi^{\rho}\left[(r+\lambda)W\right]^{1-\rho} + \frac{\lambda}{r+\lambda}\psi^{\rho}\left[(r+\lambda)Y\right]^{1-\rho}$$

for $W > Y$ *:*

$$rV_{\infty}(W) = \psi^{\rho}\left(rW+\lambda Y\right)^{1-\rho}$$

and the consumption flow is

$$c^{*}(W) = \begin{cases} \psi^{-1}(r+\lambda)W & ;\text{ for } W \in (0,Y] \\ \psi^{-1}(rW+\lambda Y) & ;\text{ for } W > Y \end{cases}$$

where $\psi = \frac{\alpha+\rho-1}{\rho} < 1$.

We know from section 2.3 that once there is a political shock, the efficient level of spending is $c^{C}(W) = (r+\lambda)W$ for $W \leq Y$ and $c^{C}(W) = rW+\lambda Y$ for $W > Y$. In the limit economy, political shocks happen at every instant, and it is not surprising that the optimal consumption level is $c^{C}(W)$. However, the lack of commitment generates inefficiencies in the limit economy in the way the assets are managed. There is too much current provision.

Corollary 1 *In limit economy, because parties cannot commit to a given* $c(W)$, *the government spends too much :* $c^*(W) > c^C(W)$.

A number of comparative statics of the limit economy are worth noting. First,

$$\frac{\partial c^*(W)}{\partial \alpha} < 0$$

As the probability of being elected to power diminishes, the parties in power spend more and save less.

Notice also that

$$\frac{\partial c^*(W)}{\partial \rho} < 0$$

As the elasticity of substitution increases $\left(\sigma = \frac{1}{\rho}\right)$, the parties consume faster out of the stock of assets. Notice that the second-best policy calls for a constant spending flow which is independent of ρ and α. These results confirm the intuition that the inefficiency that is created from the political risk is amplified when the probability of being elected is reduced and when the intertemporal elasticity of substitution is increased. In the first case, as the probability of being in power in the future decreases, the incumbent today cares less about the future and consumes at a faster rate. And, in the second case as the elasticity of substitution increases, the parties have less incentive to smooth their consumption flow, and hence will consume more today.

The parties will eventually deplete the asset stock for any $\alpha < 1$. The following proposition states this.

Proposition 7 *For any given* $w > 0$ *and* $W_t > 0$,

$$\lim_{T\to\infty} \Pr\left(\inf_{\tau\in[t,T]} \{W_\tau\} < w\right) = 1$$

The country will find itself in equilibrium with practically no assets.

I now consider the other case.

3.2 Case 2: $\alpha + \rho - 1 < 0$

The previous results on the savings equilibrium were obtained under the assumption that $\alpha + \rho - 1 > 0$. Now the case when $\alpha + \rho - 1 < 0$ is analyzed.

In this case, the limit economy as previously shown is not well defined. In particular, the previous results were based on the fact that the consumption rate converges to a finite value as γ tends to infinite. However, in the case when $\alpha + \rho - 1 < 0$ this can be shown not to be true.

Proposition 8 *If $\alpha + \rho - 1 < 0$ the consumption function $c^* (W)$ is such that*

$$\lim_{\gamma \to \infty} c^* (W) = \infty$$

As $\gamma \to \infty$ the ruling party spends faster and faster out of the government's stock of assets. The value functions converge to zero. In the limit, everything is spent in the instant a party takes power. The political risk has a dramatic impact on savings.

Proposition 9 *If $\alpha + \rho - 1 < 0$ the associated value function $V (W)$ satisfies that*

$$\lim_{\gamma \to \infty} rV (W) = 0$$

The reason why this is so is that the increase in the consumption rate lasts only for an instant dt. The assets are depleted during that instant and the provision is zero thereafter until an endowment shock arrives[13].

This proposition can be link to proposition (7). It is an extreme version of that result. In case 1, I showed that the parties will deplete the asset stock in the absence of endowment shocks. Here a similar result holds, except that it is happening at a much faster rate. The parties deplete the asset stock in a single instant.

Remark: *Even when there are no savings done in equilibrium, the parties in power* ***do not*** *put a zero discount factor on the future. They still care about the future, because there is a significant probability ($\alpha > 0$) that they might come to power again. However, in equilibrium, incumbents overspend because they expect the next instant incumbent to overspend as well, and so on. The return to savings is reduced not only because of the extra-impatience of the party in power today, but mainly because of the equilibrium behavior of future incumbents.*

[13] The party in power is consuming a stock W in a very short time. As an approximation, the utility she gets is $u\left(\frac{W}{\Delta t}\right) \Delta t$, as $\Delta t \to 0$. If the utility were bounded then it is clear that $u\left(\frac{W}{\Delta t}\right) \Delta t \to 0$. If the utility is not bounded, then by L'Hopital, we have that $\lim_{\Delta t \to 0} u\left(\frac{W}{\Delta t}\right) \Delta t = \lim_{x \to \infty} \frac{u(Wx)}{x} = \lim_{x \to \infty} u' (Wx) W$. If the Inada conditions are satisfied (which is true in our CRRA setup), we know then that $\lim_{x \to \infty} u' (x) = 0$. So, the utility over the consumption of all the wealth in an instant is zero.

4 Sustaining Stationary Promises of Repayment

We know by proposition (7) that for positive political turnover, in equilibrium the government will eventually find itself with practically no asset holdings. The parties in power would like then to borrow against the future endowment shocks. They would like to short-sell the riskless bond.

Suppose that every instant the country can issue *promises to repay* in case that the endowment shock hits. Let $X_t \in (0, Y]$ be the amount of these promises issue at time t. How much is a foreignet willing to pay for such a promise? Given the risk neutrality of the foreign investors, they are willing to pay λX_t. Under the promises, if the endowment shock happens, wealth (W_t) jumps to $Y + W_t^- - (A_t^- + X_t^-)$. If the endowment shock does not happens, then wealth under the promises evolves according to

$$\dot{W}_t = rW_t + \lambda A_t + \lambda X_t - c_t$$

A **Stationary Borrowing Contract** is a triplet $\Upsilon(W) = (A(W), c(W), X(W))$ with associate value function $V(W; \Upsilon)$.

After a default, a country can still save in the asset market, but it cannot issue promises for repayment. A Stationary Borrowing contract is sustainable if the party in power at any time prefers to maintain repay its promises, and remain in the contract, rather than defaut. The value for the party in power in case of default is the value characterized in the previous section, $V(W)$. The following definition follows,

Definition 5 *A Stationary Borrowing Contract is **sustainable** if for all W*

$$V(W; \Upsilon) > V(W) \tag{9}$$
$$V(Y + W - A(W) - X(W); \Upsilon) > V(Y + W - A(W))$$

This definition says that the party in power prefers to be in the contract all the time rather than out of it. And whenever is called to repay its promises, it prefers to do it rather than defaulting. It is assumed that the country can keep its assets after defaulting. As it will be shown, this is not going to change the results to come because the party in power has the highest temptation to default when the country has no assets.

There are many $A(W), c(W), X(W)$ that make this inequality true. For example, contracts that use the default as a trigger mechanism to sustain reputation between the political parties. I will focus on cases without this implicit reputation. For this reason, I will produce an allocation which is an equilibrium when default is not an option and later on check that under this allocation, parties have no incentive to default on their promises.

Definition 6 *A* ***Feasible Equilibrium under X(W)*** *is characterized by*

- An **asset** function $A : [0, \infty) \to (-\infty, \infty)$ with $A(W) \leq W$
- A **consumption** function $c : [0, \infty) \to [0, \infty)$ with $c(0) \in [0, \lambda X(0)]$;
  Such that $\exists V, V_0$ where:
  $c(W), A(W)$ solve

$$rV(W;X) = \max_{c, A \leq W} \{u(c) + \lambda [V(Y + W - A - X(W); X) - V(W;X)] \\ + V'(W;X)(rW + \lambda A + \lambda X(W) - c) + \gamma(1-\alpha)(V_o(W;X) - V(W;X))\}$$

and V_o is

$$rV_o(W;X) = \lambda [V_o(Y + W - A(W) - X(W); X) - V_o(W;X)] \\ + V_o'(W;X)(rW + \lambda A(W) + \lambda X(W) - c(W)) + \gamma\alpha(V(W;X) - V_o(W;X))$$

This corresponds to the previous definition of Symmetric Markov Equilibrium if $X_t = 0$. But now the possibility that the country issues promises is allowed.

The objective in this paper is to show that a country can issue promises for repayment, even in the absence of direct punishments. So, it suffices to show the existence of one contract that is sustainable. For simplicity and to be able to solve the model, I assume the following promises function.

The promises function $X(W)$ has a debt-limit D and is

$$X(W) = X(W|D) \equiv \begin{cases} D & ; \text{for } W \in [0, Y-D) \\ Y - W & ; \text{for } W \in [Y-D, Y] \\ 0 & ; \text{for } W \in (Y, \infty) \end{cases}$$

Remark: *This promises function has a simple interpretation. It is the amount of borrowing a country will get if it were facing a short-sale constraint of D and the value function were concave. However, notice that in the model, the country is not* *free to pick the amount it borrows. It is monitored by the foreigners who lend exactly $X(W)$. The foreigners observe the state of the country every instant and decide how many promises for repayment to buy.*

Rewrite the value function $V(W, X(\cdot|D))$ as $V(W|D)$.

The first order condition for consumption is $u'(c) = V'(W|D)$. And A is such that

$$\max_{A \leq W} \{\lambda V (Y + W - A - X(W|D)|D) + \lambda V'(W|D) A\}$$

If the value function is concave, the optimal portfolio decision is

$$A = \begin{cases} W & \text{; for all } W \leq Y \\ Y & \text{; for all } W > Y \end{cases}$$

I will analyze the problem as $\gamma \to \infty$ (see Appendix I). Again the analysis is separated into two cases.

4.1 Case 1

This is the case when $\alpha + \rho - 1 > 0$. The following theorem holds (see Appendix I):

Theorem 1 ***(Representation Theorem)*** *For any debt-limit D, the value function of being in power, V_∞, in the limit economy is given by*

$$V_\infty(W|D) = \begin{cases} \frac{\alpha}{r+\lambda}\Big[(1-\rho)\, u\left((r+\lambda)W + \lambda D\right) v\left(\ln \frac{(r+\lambda)W+\lambda D}{\lambda D}\right) \\ \quad + \lambda V_\infty (Y - D|D)\Big] & \text{; for } W \in [0, Y-D] \\ \frac{\alpha}{r}(1-\rho)\, u(rW + \lambda Y)\, v\left(\ln \frac{rW+\lambda Y}{\lambda D}\right) & \text{; for } W > Y - D \end{cases}$$

where v is a function such that

1. $v(0) = \frac{1}{1-\rho}$
2. $v' < 0$ *on* $(0, \infty)$
3. v *asymptotes to* $v(\infty) = \frac{\psi^\rho}{\alpha} \frac{1}{1-\rho}$.

4. for a given l, $v(l)$ is independent of λ, D, and r.

5. and $(1-\rho)v + v'$ is positive for any finite l and is increasing in l.

The dynamics under a stationary debt-limit D can be analyzed. From before it is obtained that $V'_\infty(W|D) = u'(c^*(W))$ for $W > 0$. Taking the first derivative of $V_\infty(W|D)$ with respect to W:

$$u'(c^*) = V'_\infty = \begin{cases} \alpha\left((1-\rho)v + v'\right)u'\left((r+\lambda)W + \lambda D\right) & ;\text{for } W \in (0, Y-D] \\ \alpha\left((1-\rho)v + v'\right)u'\left(rW + \lambda Y\right) & ;\text{for } W > Y-D \end{cases}$$

The sum $(1-\rho)v + v'$ is increasing in W and hence, it converges to $(1-\rho)v(\infty) = \frac{\psi\rho}{\alpha} < 1$. This implies that $(1-\rho)v + v' < 1$, for all W. So $u'(c^*) = V'_\infty < u'((r+\lambda)W + \lambda D)$ for $W \in (0, Y-D]$ and $u'(c) < u'(rW + \lambda Y)$ for $W > Y - D$. Then, consumption is always higher than $(r+\lambda)W + \lambda D$ for $W \in (0, Y-D]$ and higher than $rW + \lambda Y$ for $W > Y - D$. But $(r+\lambda)W + \lambda D$ and $rW + \lambda Y$ are the respective income flows the country gets from holding the assets, borrowing and receiving endowment shocks. The country consumes more than the income flow it receives every instant. The following then holds.

Proposition 10 *Under debt-limit D, the country consumes more than its income flow and wealth monotonically decreases towards zero in the absence of endowment shocks.*

The parties eventually consume their wealth down to zero. But if the country can borrow, once the wealth disappears the parties still have the ability to borrow against the endowment shock.

4.2 Case 2

In this case under any debt-limit D, the value function of being in power, $V_\infty(W|D)$, in the limit economy is given by

$$\lim_{\gamma\to\infty} V(W|D) = \frac{\alpha\, u(\lambda D)}{r} \tag{10}$$

The parties are unable to save, and consume all of their income in a single instant. However, they can borrow and receive a constant flow of λD. This result is again similar to Proposition 10, the striking difference is that the consumption of all the wealth is taking place now in an instant of time.

5 Sustaining Debt

In this section, I study the sustainability of debt, if the government can default on its previous promises.

5.1 Sustaining Debt, Case 1

Are the parties in power going to repay their debts? For that it is necessary to check that the parties have no incentive to default. From (9), the following has to hold

$$V_\infty (Y - D|D) > V_\infty (Y)\,; \text{ for all } W \in [0, Y - D]$$

and

$$V_\infty (W|D) > V_\infty (Y)\,; \text{ for all } W \in (Y - D, Y]$$

Where the first inequality clearly implies the second (given that V_∞ is increasing).

Using the representation theorem,

$$V_\infty (Y - D|D) = \frac{\alpha (1-\rho)}{r} u\left((r+\lambda) Y - rD\right) v \left(\ln \left(\frac{(r+\lambda) Y - rD}{\lambda D} \right) \right) \tag{11}$$

And in the case with no debt,

$$V_\infty (Y) = \frac{\alpha (1-\rho)}{r} u\left((r+\lambda) Y\right) v(\infty) \tag{12}$$

Dividing (11) by (12), the equilibrium under short-sale constraint D is sustainable if for all $W \in [0, Y]$

$$\left[\frac{u\left((r+\lambda) Y - rD\right)}{u\left((r+\lambda) Y\right)} \right] \left[\frac{v \left(\ln \left(\frac{(r+\lambda)Y - rD}{\lambda D} \right) \right)}{v(\infty)} \right] > 1 \tag{13}$$

The first term in square brackets in (13) is always less than one (for any $r > 0$) and the second is always strictly greater than one. However, as the interest rate goes down, the first term approaches one, and the second remains bounded above one for any $W \in [0, Y]$. Their product approaches a value strictly greater than one for all $W \in [0, Y]$. The following proposition follows.

Proposition 11 *For any $D \in (0, Y]$, there exists an $\bar{r} > 0$, such that for any $0 < r \leq \bar{r}$, the feasible equilibrium under debt-limit D is sustainable.*

The Bulow-Rogoff argument is not holding in this economy. Political parties repay the debt even when the credit market is as complete as the asset market and the only punishment available to the foreign investors is denial of future lending in case of default. The reason lies in the inability of the parties to save enough. Even when the parties would all like to save more, once in power they rationally choose not to. They tend to consume too much out of their asset holdings. The country eventually has very little wealth and the parties desire to borrow from the foreign creditors. If they had defaulted in the past, they won't be able to borrow again. This could be a strong enough punishment to enforce repayment of the debt. When the interest rate is low enough, parties are more patient and hence care more about the future, and the benefits of default are reduced because the return on savings is small.

How does this ability to repay relate to the political risk? The following proposition answers this question.

Proposition 12 *Let $\bar{r}(D)$ be the highest interest rate at which the feasible equilibrium under debt-limit D is sustainable; then $\bar{r}(D)$ is decreasing with α.*

As the political risk increases (α goes down) savings are more distorted. This proposition tell us that as the political risk increases, parties will repay the debts more easily (they can sustain debt contracts at a higher interest rate). However, as the political risk vanishes (as α goes to one) the following holds.

Proposition 13 ***(Bulow-Rogoff)*** *For any debt-limit D,*

$$\lim_{\alpha \to 1} \bar{r}(D) = 0$$

As α increases, the incumbent is more likely to remain in power in the future. The distortions on the savings margin are reduced and the incumbent party will find the default option more attractive. As α goes to one, a Bulow-Rogoff type of result obtains: the parties will only repay their debts if the interest rate is zero. For any positive interest rate, debt is not sustainable as an equilibrium. This proposition makes clear that the reason why parties repay the debt lies in the inefficiencies in savings that appear when the political uncertainty is high. Once the political risk vanishes and the parties are able to save more efficiently they do not need the credit market anymore and would default for any positive interest rate.

The next proposition analyzes the other extreme, when political uncertainty is high. In this case the following applies.

Proposition 14 *For any* $D \in (0, Y]$, *there exists an* $\bar{\alpha} \in (1-\rho, 1)$, *such that for any* $\alpha \in (1-\rho, \bar{\alpha}]$ *the feasible equilibrium under debt-limit* D *is sustainable.*

When the political risk is high enough (α low enough), any debt-limit D can be sustained.

Remark: (A Comparison with Harris and Laibson) *Harris and Laibson (2001) has shown that the instantaneous hyperbolic program under case 1 is equivalent (in value functions) to the program of a time-consistent consumer with a wealth-contingent utility function. However, the results on debt sustainability rely on the fact that the political parties are time-inconsistent. Both results can be reconciled once it is noticed that in the Harris and Laibson (2001) equivalence result, the wealth-contingent utility function depends on the income available in the states where* $W = 0$, *so as the amount* D *is changed, the equivalent consumer's utility function is changing. Under default, the equivalent consumer has a different utility function than under the positive short-sale constraint; and clearly the Bulow-Rogoff argument does not has to hold with a consumer that has a different utility function once he has defaulted.*

In this subsection the sustainability of debt under case one has been studied. I will now show the dramatic results that occur when $\alpha + \rho - 1 < 0$.

5.2 Sustaining Debt, Case 2

In the case with a debt-limit D (see equation (10) the following holds

$$\lim_{\gamma \to \infty} V(W|D) = \frac{\alpha\, u(\lambda D)}{r}$$

Recall from before that without debt,

$$\lim_{\gamma \to \infty} V(W) = 0$$

The following proposition is then immediate

Proposition 15 *If $\alpha + \rho - 1 < 0$ then for any $\alpha > 0$ and $D > 0$ the feasible equilibrium under debt-limit D is sustainable.*

The inefficiencies in savings created by the political risk are so large that doesn't matter what the interest rate or the elasticity of substitution are, debt would always be sustainable. Notice why: As $\gamma \to \infty$ the spending rate converges to infinity. This is the dramatic outcome of the logic : if tomorrow they (whoever are in power) are going to eat a lot, I will today eat much more. The reason why this happens is that once the savings are made, the total stock of assets belongs to the next party in power. This new incumbent will consume as much as it desires out of the total stock, making the present incumbent very reluctant to save. Debt eliminates this because once tomorrow arrives the parties hit their borrowing constraint and the dramatic logic previously exposed does not apply.

6 Autocracies versus Democracies

The previous section analyzed the sustainability of debt in a model with political turnover. One key ingredient of that model was the stability of the political parties: the parties remain in the political game forever (the value function of being out of power is not zero). I showed that as the political uncertainty increased, the ability of the parties to sustain sovereign lending increased. This was due to the inefficiencies in savings created by the political structure. I think of this political structure as representing a modern **democracy**, with several long-lived parties.

Suppose now that there is no political resurrection. Once a party is out of government, it is out forever. I call this case an **autocracy**. In an autocracy an incumbent rules continuously, but once a political shock comes and the incumbent is removed from power, she cannot return to the political game[14]. In this situation the value

[14] The case I have in mind is a dictator (and his associates) who faces exile or death once he loses power. The autocracy case is related to a system where individuals instead of parties govern, and individuals are clearly less likely to return to power than parties are.

function of an incumbent is

$$V^A(W) = E\left[\int_0^\infty e^{-(r+\gamma(1-\alpha))t} u(c_t)\, dt\right] \tag{14}$$

where $\gamma(1-\alpha)$ is the probability that the incumbent is removed from power. The political instability makes the incumbent impatient (she has an effective discount rate higher than r) but it does not make her time-inconsistent. The value function (14) is a standard exponential value function. In this case the Bulow-Rogoff result holds. The incumbent will always default on any debt contract. The model thus predicts that

- In a democracy, political turnover is positively related to debt sustainability
- In an autocracy, political turnover is not related to debt sustainability.

7 Conclusion

In this paper, I proposed a theory of sovereign debt repayment based on political economy considerations. Bulow and Rogoff (1989) show that a country that has access to a sufficiently rich asset market cannot commit to repay its debts and therefore should be unable to borrow. I show that the presence of political uncertainty reduces the ability of a country to save, and hence to replicate the original debt contract after default. In a model where different parties alternate in power, an incumbent party with a low probability of remaining in power has a high short-term discount rate and is therefore unwilling to save. The current incumbent party realizes that in the future whoever achieves power will be impatient as well, making the accumulation of assets unsustainable. Because of their inability to save, politicians demand debt ex-post and the desire to borrow again in the future enforces repayment today.

8 Appendix I : The Full Model under Case 1

In this section I derive the instantaneous system and its properties. I follow closely Harris and Laibson (2001).

The equilibrium selection technique developed by the previous authors involves three main steps. First, noise is added to the asset holdings, this guarantees that that the consumption function is continuous. It is possible to show existence in this case of a viscosity solution to the Bellman system (the reader is refered to Harris and Laibson (2001)). Second, I analyze the system as $\gamma \to \infty$, where a uniqueness result holds. And finally, I study the equilibrium as the noise vanishes.

The problem without ability to borrow is a subcase of the more general case with debt-limit D, so I will study the general problem for any value of D.

Let us add noise to the asset holdings. Both assets now evolve according to

$$dA_t = (r + \lambda) A_t dt + \sigma A_t dw_t$$
$$dB_t = rB_t dt + \sigma B_t dw_t$$

when there is no endowment shock and where w_t is a standard Brownian motion process. Notice that the process w_t is the same process for both assets. Total wealth is given by

$$W_t = A_t + B_t$$

When no endowment shock happens the wealth process evolves according to

$$dW = (rW + \lambda (A + X) - c) dt + \sigma W dw_t$$

Let us redefine the value functions. Let $J = \frac{1}{\alpha} [\alpha V + (1 - \alpha) V_0]$. Then I can write system in proposition 2 in the following way
For $W \geq 0$:

$$rV = u(c) + \lambda [V(Y + W - (A + X)) - V] + V' (rW + \lambda (A + X) - c) + \\ + \gamma (\alpha J - V) + V'' \frac{\sigma^2 W^2}{2}$$

$$rJ = u(c) + \lambda [J(Y + W - (A + X)) - J] + J' (rW + \lambda (A + X) - c) + J'' \frac{\sigma^2 W^2}{2}$$

For $W = 0$:

$$rV = u(c) + \lambda [V(Y - (A + X)) - V] + V'(\lambda(A + X) - c) + \gamma(\alpha J - V)$$
$$rJ = u(c) + \lambda [J(Y - (A + X)) - J] + J'(\lambda(A + X) - c)$$

With associated FOC:

$$u'(c) = V' \text{ ; for } W > 0 \tag{15}$$
$$c = \arg \max_{c \in [0, \lambda D]} \left\{ u(c) - V'c \right\} \text{ ; for } W = 0 \tag{16}$$

And

$$A = \arg \max_{A \in (-\infty, W + D]} \left\{ V(Y + W - (A + X)) - AV'(W) \right\}$$

Now, if the value functions were concave (I will check this later on), the optimal A is given by

$$A = \begin{cases} W & \text{; for all } W \leq Y \\ Y & \text{; for all } W > Y \end{cases}$$

Taking the limits of the Bellman system as $\gamma \to \infty$, it converges to
For $W > Y - D$:

$$-rJ + u(c) + J'(rW + \lambda Y - c) + \sigma^2 W^2 J'' = 0 \tag{17}$$

For $0 < W \leq Y - D$:

$$-rJ + u(c) + J'((r + \lambda) W + \lambda D - c) + \lambda J(Y - D) - \lambda J + \sigma^2 W^2 J'' = 0 \tag{18}$$

For $W = 0$:

$$-rJ + u(c) + J'(\lambda D - c) + \lambda J(Y - D) - \lambda J = 0 \tag{19}$$

with $\alpha J = V$. And $c \in [0, \lambda D]$ for $W = 0$.

From (15) and using the fact that $u(c) = c^{1-\rho}$; then for $W > 0$,

$$c = \left[\frac{V'}{1 - \rho} \right]^{-\frac{1}{\rho}}$$

So, $u(c) - J'c = \left[\frac{V'}{1-\rho} \right]^{\frac{\rho-1}{\rho}} - J' \left[\frac{V'}{1-\rho} \right]^{-\frac{1}{\rho}}$. Using the fact that $V' = \alpha J'$:

$$u(c) - J'c = \frac{\rho \psi}{\alpha} \left(\frac{1 - \rho}{\alpha} \right)^{\frac{1-\rho}{\rho}} J'^{\frac{\rho-1}{\rho}} \equiv h(J')$$

for $W > 0$.

For the case when $W = 0$, using (16):

$$u(c) - J'c = \begin{cases} \frac{\rho\psi}{\alpha}\left(\frac{1-\rho}{\alpha}\right)^{\frac{1-\rho}{\rho}} J'^{\frac{\rho-1}{\rho}}; \text{ if } J' > \frac{u'(\lambda D)}{\alpha} \\ (\lambda D)^{1-\rho} - J'(\lambda D) \text{ ; if } J' \leq \frac{u'(\lambda D)}{\alpha} \end{cases} \equiv h_0(J')$$

It is possible to rewrite (17) ,(18) and (19) as
For $W > Y - D$:

$$-rJ + J'(rW + \lambda Y) + h(J') + \sigma^2 W^2 J'' = 0 \tag{20}$$

For $0 < W \leq Y - D$:

$$-rJ + J'((r+\lambda)W + \lambda D) + \lambda J(Y - D) - \lambda J + h(J') + \sigma^2 W^2 J'' = 0 \tag{21}$$

For $W = 0$:

$$-rJ + J'\lambda D + \lambda J(Y - D) - \lambda J + h_0(J') = 0 \tag{22}$$

Harris and Laibson (2001) show that a system like (20) ,(21) and (22) has a unique viscosity solution J. This is the solution that will be characterized.

8.0.1 Proving Concavity

Let us check now that the viscosity solution J that solves (20), (21) and (22) is concave.

Suppose now that $J'' = 0$. Taking first derivatives (from both equations (17) and (18)): $J'''\sigma^2 W^2 = [J' - u'(c)]\,c'$. But when $J'' = 0$,from the FOC of consumption we have that $\alpha J' = V' = u'(c) \Rightarrow \alpha J'' = u''(c)\,c' \Rightarrow c' = 0$. So $J''' = 0$ whenever $J'' = 0$. If there is any W_1 such that $J''(W_1) \geq 0$, this implies that $J''(W_t) \geq 0$, for all $W_t > W_1$. Then, J grows at least linearly for any $W > W_1$, which contradicts the CRRA (boundness) assumption.

So $J''(W)$ cannot be non-negative, for any $W > 0$. The value function in the limit is concave and A is optimal. It is also possible to show uniqueness of this solution as the instantaneous hyperbolic program (17) , (18) and (19) can be rewritten as the program of a time-consistent consumer with a wealth-dependent utility function. See Harris and Laibson (2001).

8.0.2 The Viscosity Solution When The Noise Vanishes

I now let the noise vanish. In particular as $\sigma^2 \to 0$, the value functions $J(\sigma^2)$ uniformly converge on compact subsets of $[0, \infty)$ to the unique viscosity solution of the following

system :
For $W > Y - D$:

$$-rJ + J'(rW + \lambda Y) + h(J') = 0 \tag{23}$$

For $0 < W \leq Y - D$:

$$-rJ + J'((r+\lambda)W + \lambda D) + \lambda J(Y - D) - \lambda J + h(J') = 0 \tag{24}$$

For $W = 0$:

$$-rJ + J'(\lambda D) + \lambda J(Y - D) - \lambda J + h_0(J') = 0 \tag{25}$$

Let v be define as the following

$$v(l) = \begin{cases} (r+\lambda)\dfrac{J\left(\dfrac{\exp(l + \ln(\lambda D)) - \lambda D}{r+\lambda}\right) - \dfrac{\lambda}{r+\lambda}J(Y-D)}{(1-\rho)\,u(\exp(l+\ln(\lambda D)))} \\ \qquad ;\ \text{for } l \in [0, \ln((r+\lambda)Y - rD) - \ln(\lambda D)] \\ r\dfrac{J\left(\dfrac{\exp(l+\ln(\lambda D)) - \lambda Y}{r}\right)}{(1-\rho)\,u(\exp(l+\ln(\lambda D)))} \\ \qquad ;\ \text{for } l > \ln((r+\lambda)Y - rD) - \ln(\lambda D) \end{cases} \tag{26}$$

Substituting v into the system, we have that J satisfies (23), (24) and (25) iff v satisfies
For $l > 0$:

$$\left((1-\rho)v + v'\right) - v + (1-\rho)^{-\frac{1}{\rho}} h\left((1-\rho)v + v'\right) = 0 \tag{27}$$

For $l = 0$:

$$\left((1-\rho)v + v'\right) - v + (1-\rho)^{-\frac{1}{\rho}} h_0\left((1-\rho)v + v'\right) = 0 \tag{28}$$

The advantage of this result is that this new system, (27) and (28) , is independent of wealth W. I can then show that there exists a smooth function $H(\cdot)$ such that $v = H(v')$.

It is possible to show the following

- $H'' > 0$
- $H'(0) = 0$
- $\min H = H(0) = \frac{\psi^\rho}{(1-\rho)\alpha} < \frac{1}{1-\rho}$

Let $v_0 = \frac{1}{1-\rho}$; (here v_0 corresponds to the value when a country has no wealth, endowment shocks never happen, and the parties consume always λD). The unique viscosity solution $v(l)$ to the system (27) and (28) is such that

- $v(0) = v_0$
- $v' < 0$ on $(0,\infty)$
- v assymptotes to $H(0) = v(\infty)$.
- for a given l, $v(l)$ is independent of λ, D, and r.
- and $(1-\rho)v + v'$ is positive for any finite l and is increasing in l.

The following figure plots the function H in the (v', v) space.

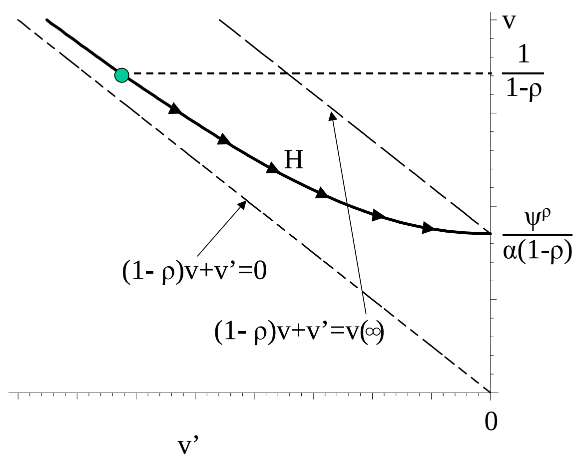

Figure 1-1: The function H

Figure 1.2 shows the graph of v as a function of l.

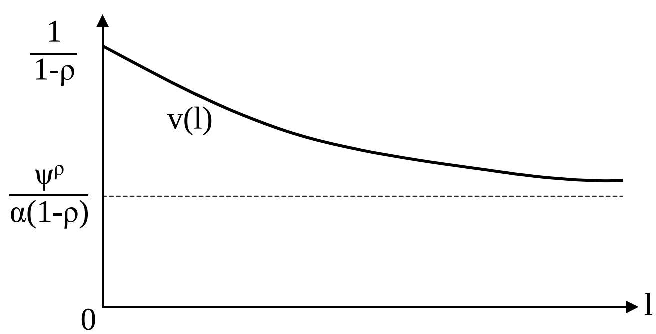

Figure 1-2: The function v(l)

8.0.3 The Representation Theorem

Going back to the original system and using (26), I have that J can be represented as

$$J(W) = \begin{cases} \frac{1}{r+\lambda}\left[(1-\rho)\,u\left((r+\lambda)\,W+\lambda D\right)v\left(\ln\left((r+\lambda)\,W+\lambda D\right)-\ln\left(\lambda D\right)\right)\right. \\ \left.+\lambda J\left(Y-D\right)\right] \qquad ; \text{ for } W \leq Y-D \\ \\ \frac{1}{r}\left(1-\rho\right)u\left(rW+\lambda Y\right)v\left(\ln\left(rW+\lambda Y\right)-\ln\left(\lambda D\right)\right) \\ \qquad ; \text{ for } W > Y-D \end{cases}$$

This is the representation theorem.

I can also compute the value when there is no debt. When $D \to 0$, the solution converges to $H(0)$. The value function is given by

$$J(W) = \begin{cases} \frac{1}{r+\lambda}\left[(1-\rho)\,u\left((r+\lambda)\,W\right)v\left(\infty\right)+\lambda J\left(Y\right)\right] \\ \qquad ; \text{ for } W \leq Y \\ \frac{1}{r}\left(1-\rho\right)u\left(rW+\lambda Y\right)v\left(\infty\right) \\ \qquad ; \text{ for } W > Y \end{cases}$$

For any D, the associated (limit) value function J is concave. This is just because J is obtained as the limit of concave functions (as noise vanishes).

9 Appendix II: Proofs

Proof Proposition (3): For $W \leq Y$, let $V(W) = \phi W^{1-\rho} + \beta$ and $V_o(W) = \phi' W^{1-\rho} + \beta'$. Substituting in the assumed value functions, letting $\mu = c/W$, and using the FOC for consumption, it is obtained that $\mu^{-\rho} = \phi$. Solving for the value function

$$\phi = \frac{\mu^{1-\rho}}{(r+\lambda)\rho + (1-\rho)\mu}\left[\frac{(r+\lambda)\rho + (1-\rho)\mu + \gamma\alpha}{(r+\lambda)\rho + (1-\rho)\mu + \gamma}\right]$$

$$r\beta = \left(\frac{(r+\lambda)\rho + (1-\rho)\mu + \gamma\alpha}{(r+\lambda)\rho + (1-\rho)\mu} - \frac{\gamma(1-\alpha)}{r+\gamma}\right)\frac{\lambda\mu^{1-\rho}Y^{1-\rho}}{(r+\lambda)\rho + (1-\rho)\mu + \gamma}$$

Let μ^* be such that $F(\mu^*) = 0$ where $F(\cdot)$ is obtained from the FOC:

$$F(\mu) = \frac{1-\rho}{r+\lambda}\mu^2 + \left(2\rho - 1 + \frac{\gamma}{r+\lambda}\frac{\alpha+\rho-1}{\rho}\right)\mu - ((r+\lambda)\rho + \gamma) \tag{29}$$

It is easy to see that $F(0) < 0$ and it is a parabola that opens up, so it has a unique positive root. Taking the derivative of the implicit function $F'(\mu) = 2\frac{(1-\rho)}{r+\lambda}\mu + 2\rho - 1 + \frac{\gamma}{r+\lambda}\frac{(\alpha-(1-\rho))}{\rho}$; which evaluated at $r+\lambda$ yields $F'(r+\lambda) = 1 + \frac{\gamma}{r+\lambda}\frac{(\alpha-(1-\rho))}{\rho} > 0$. Evaluating the implicit function at $r+\lambda$ I obtain that $F(\mu^{SB}) = \gamma\frac{\alpha-(1-\rho)}{\rho} - \gamma$ which implies that $F(\mu^{SB}) < 0$. Given that $F(r+\lambda) < 0$ and $F'(r+\lambda) > 0$, then μ^* that solves $F(\mu^*) = 0$ is such that $\mu^* > (r+\lambda)$ ■

Proof Proposition (4): For $\frac{d\mu^*}{d\gamma} > 0$: Differentiating the implicit function

$$\frac{d\mu^*}{d\gamma} = \frac{1 - \mu^*\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}}{\frac{2(1-\rho)}{r+\lambda}\mu^* + 2\rho - 1 + \gamma\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}}$$

Given that $\mu^* > r+\lambda$, the denominator is always positive. We know that $\mu^* \to r+\lambda$ as $\gamma \to 0$ and $\mu^* \to \left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}$ as $\gamma \to \infty$. That means that if there exists $\gamma' < \infty$ such that $\mu^*(\gamma') > \left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}$, there has to exists a $\gamma'' < \infty$ such that $\mu^*(\gamma'') = \left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}$. Now, $F\left(\left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}\right) = 0$ implies that $\frac{1-\rho}{r+\lambda}\frac{(r+\lambda)^2\rho^2}{(\alpha+\rho-1)^2} + 2\rho\frac{(r+\lambda)\rho}{(\alpha+\rho-1)} - (r+\lambda)\rho = 0$. This equation is not a function of γ and for any $\alpha < 1$, this equation does not hold. There is no $\gamma'' < \infty$ such that $\mu^*(\gamma'') = \left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}$ and hence $\mu^* < \left(\frac{(\alpha+\rho-1)}{(r+\lambda)\rho}\right)^{-1}$ for any γ. This implies that the numerator is always positive.

For $\frac{d\mu^*}{d\alpha} < 0$: Differentiating the implicit function

$$d\mu = -\frac{\frac{\gamma}{r+\lambda}\frac{1}{\rho}\mu}{2\frac{1-\rho}{r+\lambda}\mu + \left(2\rho - 1 + \frac{\gamma}{r+\lambda}\frac{(\alpha-(1-\rho))}{\rho}\right)}d\alpha$$

The numerator is positive and the denominator has been previously shown to be positive. This completes the proof.■

Proof of proposition (5): See previous Appendix I.■

Proof of proposition (6): From previous Appendix I, the representation theorem implies that

$$J(W) = \begin{cases} \frac{1}{r+\lambda}\left[(1-\rho)\,u\left((r+\lambda)\,W\right)v(\infty) + \lambda J(Y)\right] & ;\ \text{for } W \leq Y \\ \frac{1}{r}(1-\rho)\,u(rW + \lambda Y)\,v(\infty) & ;\ \text{for } W > Y \end{cases}$$

Substituting into for $v(\infty) = \frac{\psi^\rho}{\alpha(1-\rho)}$ and $u(c) = c^{1-\rho}$ we can obtain the value function $V_\infty = \alpha J$. Taking the first derivative of V_∞ with respect to W we have that $u'(c) = V'(W)$. We can then solve for $c^*(W) = \frac{(V')^{-\rho}}{1-\rho}$. And this gives us the equilibrium consumption rule. ■

Proof of corollary (1): It is clear as $\psi < 1$.■

Proof of proposition (7): Given that in the $c(W)$ is higher than $(r+\lambda)W$ when $W \leq Y$ and higher than $rW + rY$ when $W > Y$, wealth is monotonically decreasing to zero in the absence of endowment shocks. This implies that for any initial W_0, the wealth process eventually converges to $[0, Y]$. For any $W_t \in (k, Y]$, there exists a $T < \infty$ such that if no endowment shock happens, $W_{t+T} \leq k$. So I need the endowment shock not to happen in an interval of size T. This is a positive probability event. Given that time is infinite, it will happen with probability one.■

Proof of proposition (8): See Harris and Laibson (2001)■

Proof of proposition (9): See Harris and Laibson (2001)■

Proof of theorem (1): See previous Appendix I.■

Proof of proposition (10): The first part is proven in the text. The second part follows inmediately.■

Proof of proposition (11): In the text.■

Proof of proposition (12): Suppose that a contract D is just sustainable for some $r > 0$. This implies that $\left[\frac{v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)}{v(\infty)}\right]\left[\frac{u(r+\lambda)Y-rD}{u((r+\lambda)Y)}\right] = 1$. Now, as α increases, $v(\infty)$

increases, and $v(l)$ moves down to $v(\infty)$. This implies that $\frac{v(l)}{v(\infty)}(>1)$ decreases with α. And the previous equality breaks. So r can not sustain the contract anymore. This implies then that $\bar{r}(D)$ has been reduced with the increase in α.■

Proof of proposition (13): As $\alpha \to 1$, $v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)$ converges to $\frac{1}{1-\rho}$. This implies that $\frac{v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)}{v(\infty)}$ converges to 1 as $\alpha \to 1$. Given that for any $r > 0$, $\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} < 1$, then for some α close to 1, $\left[\frac{v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)}{v(\infty)}\right]\left[\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)}\right] < 1$. I can do that for all $r > 0$, so as $\alpha \to 1$, only $r = 0$ is sustainable.■

Proof of proposition (14): It is easy to see that as $\alpha \to 1-\rho$, $v(\infty) \to 0$ and $v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)$ remains bounded below by a strictly positive value. This implies that $\left[\frac{v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)}{v(\infty)}\right]$ goes to infinity as $\alpha \to (1-\rho)$, so for any r, I can find an $\bar{\alpha}$ such that for any $\alpha \in (1-\rho, \bar{\alpha}]$, $\left[\frac{v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)}{v(\infty)}\right]\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} > 1$; which proves the proposition■

Proof of proposition (15): Immediate because $\lim V = 0 < \lim V^D$ for any $D > 0$ as long as $\alpha > 0$.■

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