

# SOVEREIGN DEBT AND THE TRAGEDY OF THE COMMONS\*

Manuel Amador<sup>†</sup>  
Stanford University

July 23, 2012

## Abstract

In this paper I study a political economy model of a small open economy composed of different groups that compete for access to government resources. The government can save and borrow from risk-neutral foreigners. If the government defaults on its sovereign debt, it can be excluded from future borrowing, but not from lending. No other punishments are imposed. I show that the same political economy forces that generate overspending can also guarantee that the small open economy repays its sovereign obligations, showing that [Bulow and Rogoff \(1989\)](#)'s result does not hold: countries repay because they would like to borrow again.

(JEL codes: H63, P16)

---

\*The ideas presented here are based on the first chapter of my dissertation. The author would like to thank seminar participants at different universities and institutions for their feedback. Luke Stein provided excellent research assistance. Daron Acemoglu, Mark Aguiar, Marios Angeletos, Doireann Fitzgerald, Martin Schneider, Jaume Ventura, Ivan Werning and Mark Wright provided valuable advice. This research has been supported with a grant of the National Science Foundation.

<sup>†</sup>Email: amador@stanford.edu. Address: 579 Serra Mall, room 330, Stanford, CA 94305-6072

# 1 Introduction

Emerging markets borrow substantial amounts of foreign debt. Most of what they borrow, they eventually repay, even though there are no clear punishments available to creditors besides the threat of eliminating future lending (Eaton and Fernandez, 1995). The absence of clear punishments raises the question of why countries repay their debts.

One of the oldest reasons for repayment of sovereign obligations is the well-known reputational argument: countries choose to repay their debts because they would like to borrow again in the future.<sup>1</sup> This argument was first formalized in the economics literature by Eaton and Gersovitz (1981) in the context of a small open economy subject to endowment shocks: by defaulting, the country loses access to international credit markets and is not be able to smooth its expenditures<sup>2</sup>. The resulting variation in expenditures can be painful enough to enforce repayment.<sup>3</sup> The fact that direct sanctions after default are no longer observed in the data makes the reputational argument a very compelling one. Indeed, in his historical analysis of the American State Debts of the 1840s, English (1996) reaches the conclusion that, in spite the lack of sanctions, most states repaid their debts in full in order to maintain access to capital markets, as predicted by the reputational model.

However, in an important theoretical contribution, Bulow and Rogoff (1989) demonstrated that the reputational argument for repayment breaks down when countries can save after default: by using the asset markets to self-insure, a country can always guarantee higher consumption if it defaults than if it does not. Given the richness of the international financial markets, it is unrealistic to assume that defaulting countries cannot save; and hence a repayment paradox emerges.

The present paper provides an answer to why countries repay their debts in the absence of sanctions. It does so by changing an implicit assumption of the literature on sovereign debt: that a country's decision to default can be modelled as that of a representative agent. The paper breaks with this tradition by recognizing that the repayment of sovereign debt is not only an economic decision but also a political one.

I set up a simple political economy model in which a small open economy is composed of long-lived political groups with distinct interests. The political groups share common

---

<sup>1</sup>Alexander Hamilton, when founding the Treasury Department and consolidating the states debt into a federal one, used the reputational argument to support his view that the external debt accumulated during war of independence should be repaid (see footnote 12 of English, 1996).

<sup>2</sup>Several authors have extended the reputation approach to sovereign lending. See for example Atkeson (1991); Grossman and Van Huyck (1988); Worrall (1990).

<sup>3</sup>A recent literature is trying with some success to seriously quantify the implications of Eaton and Gersovitz (1981) model (Aguiar and Gopinath, 2006; Arellano, 2008). In these recent contributions, however, sovereign debt is enforced by an external punishment that is not modeled.

access to a savings technology that is controlled by a government, generating a tragedy of the commons problem regarding the country's assets. Models of this type have been used to explain why emerging markets governments over-spend and follow pro-cyclical policies (see [Tornell and Velasco, 1992](#); [Tornell and Lane, 1999](#)): domestic groups demand too much spending from the government because they enjoy the benefits privately but all share the costs through the government budget constraint. The main contribution of this paper is to show that the same arguments that lead to over-spending in a political model also provide the incentives for repayment of sovereign obligations. A political economy model can thus reconcile two seemingly contradictory observations: emerging markets' politicians do not save and spend too much and most of the time they pay back their debts even in the absence of sanctions.

In the model I put forward in this paper, the small open economy has access to assets after default, and a modified version of the theorem by [Bulow and Rogoff \(1989\)](#) holds: after defaulting, there exists an allocation generated only with savings that Pareto improves over the repayment of the debt. Usually, the existence of this default allocation is enough for one to argue that a domestic government would prefer it and default will always ensue. However, in a political game, where multiple groups inside the country are part of the decision making, the allocation after default must also be the result of a political equilibrium. As I will show, this is not generally the case, even when the long-lived domestic groups could in principle cooperate after defaulting.

The political inability of a country to maintain a buffer stock of assets for self-insurance is a critical part of the argument for repayment developed in this paper. In several historical episodes, the accumulation of budget surpluses has been shown to be politically impossible. [Cole et al. \(1995\)](#) and [English \(1996\)](#) describe the case of the American States in the 1830s. At that time, the accumulation of a large federal surplus was controversial and at the end, the surplus was distributed to the states. The states did not hold the money for long, and spent or distributed it. A few years later, Benjamin R. Curtis, a supreme court judge, specifically argued that a state's reputation in credit markets was important because American states could not accumulate surpluses, and in an emergency they might need more resources than they could tax in a single year.<sup>4</sup>

However, the above discussion misses an important point: how come the political parties that make the accumulation of surpluses impossible can instead agree on the repayment of debt? In my model, the domestic groups realize that if they were to default they would

---

<sup>4</sup>See page 271 of [English \(1996\)](#). [English \(1996\)](#) goes on to say that "The reason for the inability of states to accumulate surpluses is not given by Curtis, but it presumably was the result of U.S. citizens' distrust of government ... If citizens believed that state officials will either steal or waste a large enough share of the surplus, then they will be willing to give up the gain to be had by repudiation."

inefficiently overspend and save too little even though they would all benefit from saving more. Importantly, the sustainability of a buffer stock can be infeasible not just because the domestic groups do not agree to save today, but also because they might not agree to save in the future. The argument for repayment arises from showing that these dynamic inefficiencies can be reduced with appropriately designed access to external credit: by granting access to funds only in states where the country really needs them, while curtailing them in times when they are not so needed, international financial markets in effect provide a commitment technology that can be preferred to default. Note that it is in the interest of the creditors to restrict access to credit in this manner, as this is the fundamental feature of a debt contract that guarantee its repayment.

## Related Literature

There is an extensive empirical literature documenting that political instability, corruption, and weak property rights go hand-in-hand with lower savings. For example, in their analysis of international reserve-holding behavior by developing countries, [Aizenman and Marion \(2004\)](#) show that countries with higher indexes of political corruption tend to accumulate lower levels of reserves. They argue that political corruption reduces the size of the buffer stocks held by a government by increasing opportunistic behavior. Regarding the dissipation of positive income shocks, [Tornell and Lane \(1999\)](#) document that when Nigeria, Venezuela, Mexico, Costa Rica, Cote d'Ivoire and Kenya received significant windfalls from their terms of trade, their respective governments increased spending more than proportionally to the increased revenue. [Easterly and Levine \(1997\)](#) provide evidence that linguistic diversity in African countries, a proxy for political fragmentation, is correlated with a range of bad public policies such as low education and little provision of infrastructure.

Several explanations of why countries repay their debts have been proposed in the literature. Researchers have studied the possibility that reputation spillovers to other valuable relationships might be costly enough to enforce repayment ([Cole and Kehoe, 1995](#) and [Cole and Kehoe, 1997](#))<sup>5</sup>. Another approach looks at the assets available to the country after default: technological restrictions ([Kletzer and Wright, 2000](#)) or collusion among banks ([Wright, 2004](#)) might reduce the range of savings mechanisms available to the country after default. Another branch of the literature studies the punishments available to creditors,

---

<sup>5</sup>For example, [Aguiar et al. \(forthcoming\)](#) study the dynamics of sovereign debt in a small open economy model with capital accumulation. Debt is sustainable because government default increases the expected expropriation rate and reduces domestic investment. See also [Aguiar and Amador \(2011\)](#).

from military intervention to trade embargoes.<sup>6,7</sup>

The basic model of this paper is based on the dynamic tragedy of the commons models studied by [Tornell and Velasco \(1992\)](#) and [Tornell and Lane \(1999\)](#). The repayment of international debt, the issue of interest here, is not a concern in those papers. Also, they analyze only Markov-perfect equilibria. In this paper, I instead study sub-game perfect equilibria of a dynamic game and provide a full characterization of the efficient equilibria with symmetric payoffs. As will be argued later on, this characterization of the efficient equilibria is necessary to convincingly answer the question of repayment. A related paper is [Svensson \(2000\)](#). He also studies a repeated tragedy of the commons model, focusing on the effects of foreign aid on the collusive behavior of political groups, again a different question from the one I address here.

There is a broad literature on the political economy of fiscal deficits, initiated by [Tabellini and Alesina \(1990\)](#) and [Persson and Svensson \(1989\)](#). In these early papers, the possibility of default on government debt is not considered. [Tabellini \(1991\)](#) and [Dixit and Londregan \(2000\)](#) present models of sustainability of domestic debt. In their models, the lenders are citizens and thus have political rights (they can vote). Here, I instead analyze a model of *sovereign* debt, where lenders reside outside the country and have no political rights.<sup>8</sup>

My results are also related to the work by [Gul and Pesendorfer \(2004\)](#) who develop a theory of preferences for commitment. They study a consumer with these preferences and show how the [Bulow and Rogoff \(1989\)](#) result could be overturned. In contrast, in this paper I study a fully specified political economy model, and the results regarding debt sustainability relate to political variables.

The layout of the paper is as follows: Section 2 sets up the model without debt, and proceeds to characterize the best equilibrium. This will be the equilibrium that a country will be assumed to revert to after default. Section 4 introduces the possibility of borrowing from foreigners and studies the issue of repayment. I show that the argument of [Bulow and Rogoff \(1989\)](#) does not hold in general except when there is only one group, or the groups are infinitely patient. All proofs are collected in the appendix.

---

<sup>6</sup>[Rose \(2005\)](#) has shown that after a country defaults, its international trade is significantly reduced, identifying a channel through which external creditors might be punishing the defaulting country.

<sup>7</sup>[Sandleris \(2005\)](#) presents a different argument for repayment, based on asymmetric information.

<sup>8</sup>There are several models of political economy analyzing efficient subgame perfect equilibrium, just as I do in this paper. See for example the work of [Dixit et al. \(2000\)](#) (who study efficient equilibria in the [Alesina \(1988\)](#)'s political turnover model), the more recent work of [Acemoglu et al. \(2008\)](#) with self-interested politicians.

## 2 Set Up of the Model

Time is discrete and runs from 0 to infinity. There is a small open economy, composed of  $n$  infinitely-lived domestic groups indexed by  $i \in I = \{1, 2, \dots, n\}$ , and a government.

The government receives an endowment  $e_t$  at the beginning of every period  $t$ . Between periods, the government can access an international financial market for both saving and, if it has not previously defaulted, borrowing. The international financial market is assumed to be populated by agents willing to trade consumption inter-temporally at a constant (gross) interest rate of  $R$ .

At the end of every period, group  $i$  receives a fiscal provision from the government, denoted by  $d_i$ . As of time  $t = 0$ , the utility to group  $i$  generated from a sequence  $\{d_t^i\}_{t=0}^\infty$  of fiscal provisions is given by

$$U^i(\{d_t^i\}) = \sum_{t=0}^{\infty} \beta^t u_t^i(d_t^i),$$

where  $\beta$  is the discount factor, assumed to be the same across all the groups.

The objective of the paper is to understand how political economy distortions, combined with a recurrent desire to trade inter-temporally, affects debt repayment. Because of this, I now make a couple of stylized assumptions so that the desire to shift resources across time is starkly transparent. In particular, I narrow attention to a deterministic environment, as the issue of repayment of debt relies on the desire to trade inter-temporally (and not on risk-sharing).

So as to facilitate the analysis that follows, I also assume that the utility of the groups is linear. That is,  $u_t^i(d) = \phi_t d$ . To generate a desire to trade inter-temporally, I assume that the aggregate values of  $\phi_t$  and  $e_t$  behave according to the following deterministic rules:

$$\phi_t = \begin{cases} 1 & ; \text{for } t \in T_1 \\ \phi & ; \text{for } t \in T_\phi \\ \phi^2 & ; \text{for } t \in T_{\phi^2} \end{cases}$$

and

$$e_t = \begin{cases} 1 & ; \text{for } t \in T_1 \\ 0 & ; \text{otherwise} \end{cases}$$

for some parameter  $\phi > 1$ , and where  $T_{\phi^2} = \{t | t \equiv 0 \pmod{3}\}$ ,  $T_1 = \{t | t \equiv 1 \pmod{3}\}$  and  $T_\phi = \{t | t \equiv 2 \pmod{3}\}$ . That is, the endowment sequence,  $\{e_t\}_{t=0}^\infty$ , repeats itself every three

periods and equals  $\{0, 1, 0, 0, 1, 0, 0, \dots\}$ , while the sequence of aggregate utility parameters,  $\{\phi_t\}_{t=1}^\infty$ , also repeats every three periods and equals  $\{\phi^2, 1, \phi, \phi^2, 1, \phi, \phi^2, \dots\}$ .

Note that a desire to trade inter-temporally arises in this linear model through the negative correlation between  $\phi_t$  and  $e_t$ : the country is receiving the endowment in those periods where marginal utility (to all groups) is the lowest. Note also that the aggregate parameter  $\phi_t$  captures the benefit to domestic groups of government provision at any time  $t$ . That is, all groups agree on the periods in which government spending is relatively more valuable. Hence, the parameter  $\phi_t$  is a catch-all for events that affect all groups in the country in a similar fashion: it can represent a natural disaster, a low aggregate productivity state, or a war.

The following restrictions on  $\beta$  and  $R$  are made to bound the payoffs and to guarantee that saving the endowment for better times is efficient:

**Assumption 1.** *The parameters  $\beta$ ,  $\phi$  and  $R$  are such that (i)  $\beta \in (0, 1)$ ,  $R > 1$ ; (ii)  $\beta R < 1$ ; and (iii)  $\beta R \phi > 1$ .*

Part (ii) imposes that the country be more impatient than the foreigners. This guarantees that it would not be efficient for the country to save without bounds. On the other hand, part (iii) guarantees that, if the country has any resources available, it would be efficient to save from a period where marginal utility is low to a following period with higher marginal utility.

A fiscal allocation is defined to be a non-negative and non-stochastic sequence of fiscal provision vectors to each group,  $d = \{(d_t^1, \dots, d_t^n)\}_{t=0}^\infty$ .

If the country could commit to repay its debts, then, given Assumption 1, any Pareto optimal fiscal allocation will feature the country borrowing in period  $t = 0$  and spending resources only in that period, while paying back the entire endowment process to the international financial markets in future periods. Clearly, such allocation may not be attainable if the groups can choose to eventually default.

I assume, as in [Bulow and Rogoff \(1989\)](#), that the foreign investors can commit to exclude the country from borrowing again in foreign financial markets if the country ever defaults on its sovereign debt; but they cannot stop the country from saving after default. To understand the sustainability of sovereign debt, it is necessarily to first characterize the equilibria of the game after default. This is what I do in the next section.

### 3 The Equilibria After Default

Consider then the case, as of time  $t_0$ , where the government has defaulted in the past (hence, it cannot longer borrow from abroad) and has an initial asset position equal to  $a_{t_0-1} \geq 0$ .

Let  $a(d) = \{a_{t-1}(d)\}_{t=t_0}^{\infty}$  be the sequence of assets positions generated by a fiscal allocation  $d$  with initial assets  $a_{t_0-1}$ , that is:

$$a_t(d) = Ra_{t-1}(d) + e_t - \sum_{i=1}^n d_t^i \quad (1)$$

for all  $t \geq t_0$ . Then, an allocation is *feasible after default* if the asset positions generated by it are non-negative for all  $t \geq t_0$ :

**Definition 1.** *An allocation  $d$  is **feasible after default** if the asset positions generated by  $d$  are non-negative, that is,  $a_t(d) \geq 0$  for all  $t \geq t_0$ .*

Given Assumption 1, it follows that, if groups could commit, they will save all of their wealth in periods  $T_1$  and  $T_\phi$ , that is, periods when marginal utility is low. They will consume their wealth in the subsequent periods  $T_{\phi^2}$ , when marginal utility is the highest. Let me denote such saving behavior as “first best”. Whether first best behavior arises in equilibrium depends on the structure of the game, which I proceed to describe next.

The political game is modelled as follows: At the beginning of every period, each group  $i$  makes a fiscal demand to the government,  $\hat{d}_t^i$  and, in return, receives a fiscal provision from the government,  $d_t^i$ .<sup>9</sup> The government’s objective is just to satisfy the current demands of all the groups whenever this is feasible. That is,  $Ra_{t-1} + e_t$  denotes the amount of resources available to the government in period  $t$ . If  $Ra_{t-1} + e_t \geq \sum_{i=1}^n \hat{d}_t^i$ , then provisions equal demands,  $d_t^i = \hat{d}_t^i$ . On the other hand, if the sum of fiscal demands at time  $t$  exceeds the available resources, that is  $\sum_{i=1}^n \hat{d}_t^i > Ra_{t-1} + e_t$ , then the government cannot provide each group with its desired provision, and I assume that each group, in this case, receives an equal share of the maximum spending possible:  $d_t^i = (Ra_{t-1} + e_t)/n$ .

Although specific, this particular political game tries to capture important features of more general political processes such as (a) that the resources in the hands of the government are, in principle, available to all groups; (b) that the groups cannot ex-ante commit to any division of resources; and finally (c) that, given the public nature of the government resources, a group must be concerned the others might appropriate a large fraction of the fiscal budget.<sup>10</sup>

<sup>9</sup>This political game is a version of that used by [Tornell and Velasco \(1992\)](#) and [Tornell and Lane \(1999\)](#).

<sup>10</sup>The particular tie breaking rule used for the case when all groups demand too much can be alternatively justified as a form of government breakdown. For example, it could be assumed that, at a cost, a group can guarantee equal sharing of all the resources of the country. As will be shown in what follows, the focus on



It is possible to characterize the worst equilibrium of this game. Let the total-exhaustion strategy profile be given by infinite fiscal demands,  $\hat{d}_t^i = \infty$ , after any history for all groups. Then the following holds,

**Lemma 1** (Worst Equilibrium). *The total-exhaustion strategy profile is a subgame perfect equilibrium of the game after default. Even more, after any history, this strategy profile generates the lowest possible equilibrium payoff to all groups.*

In the worst equilibrium, there is no cooperation among the groups because of their fears that what is not demanded will be consumed by the other groups.

Subgame-perfect allocations can now be characterized as feasible allocations after default supported by the threat of using the worst equilibrium payoff as a continuation value:

**Definition 2.** *A feasible allocation after default,  $d$ , is **subgame-perfect** if it satisfies the following sustainability constraints:*

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i \geq \phi_t \max \left\{ Ra_{t-1}(d) + e_t - \sum_{j \neq i} d_t^j, \frac{Ra_{t-1}(d) + e_t}{n} \right\} + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \phi_{\tau} \frac{e_{\tau}}{n}, \quad (\text{SC})$$

for all  $t \geq t_0$  and  $i \in I$ .

The constraints (SC) ensure that no group prefers to deviate from the prescribed allocation when facing the worst equilibrium payoff as continuation value. The first term of the right hand side of the above equation describes that when a group deviates, it can guarantee itself a flow payoff that is at most equal to the maximum between forcing equal shares of the resources in the current period, or demanding all of the residual resources. The second term of the right hand side is the discounted worst equilibrium payoff which is used as a continuation value.

### 3.1 An Efficient And Symmetric Subgame-perfect Allocation

The final step in characterizing the game after default is to determine the maximal amount of domestic cooperation that can be sustained after a default episode. With this goal in mind, I define efficient allocations as follows:

**Definition 3.** *A feasible allocation after default is **efficient** if it is subgame-perfect and there does not exist another subgame-perfect allocation after default that weakly Pareto dominates it as of time  $t_0$ .*

---

equilibria with symmetric payoffs reduces the importance of the tie-breaking rule, because a deviating group would never desire to trigger it.

From the symmetry in the structure of the game, it is possible to show that there exists an efficient allocation that is symmetric: that is, that all fiscal demands in equilibrium are the same for all groups. I will later show that this symmetric efficient allocation contains all the information required to study the sustainability of sovereign borrowing.

Note that in a symmetric allocation, the best deviation by any given group is not to force equal sharing of the government resources but rather to demand all of the residual resources.<sup>11</sup> I can then show that an efficient and symmetric allocation after default features no savings in periods of highest marginal utility (that is,  $a_t = 0$  for  $t \in T_\phi^2$ ); and that, after the first time  $\hat{t} > t_0$  such that  $\hat{t} \in T_\phi^2$ , the efficient allocation is 3-period stationary. The next lemma summarizes these results:

**Lemma 2** (Symmetry, No Over-saving and Stationarity). *Suppose Assumptions 1 holds. Then, there exists an efficient allocation after default,  $d$ , that is symmetric and features no over-saving: that is,  $a_t(d) = 0$  for all  $t \in T_\phi^2$  and  $t \geq t_0$ . In addition, let  $\hat{t}(t_0) = \min\{\tau \geq t_0 : \tau \in T_\phi^2\}$ , then such efficient allocation is three period stationary after  $\hat{t}(t_0)$ : that is,  $d_t^i = d_{t'}^j$  for all  $i, j \in I$  and  $t \equiv t' \pmod{3}$  with  $t, t' > \hat{t}(t_0)$ .*

In a symmetric allocation, the fiscal provisions to each group are identical, and hence, to characterize the allocation, it suffices to know the equilibrium savings, as the provisions will be determined by the budget constraint (1). Lemma 2 significantly reduces the set of possible efficient allocations, as we just need to consider ones with no saving in  $T_\phi^2$  periods and that are three period stationary after  $\hat{t}(t_0)$ . This Lemma can be used to provide a characterization of the efficient and symmetric equilibrium, but before doing this, let me first state some definitions that will be used in what follows.

**Definition 4.** *Let  $\theta = \beta R\phi$ . Let  $(\hat{a}_1, \hat{a}_\phi)$  be as follows:*

$$(\hat{a}_1, \hat{a}_\phi) = \begin{cases} (0, 0) & ; \text{if } (\beta, n, \theta) \in \Gamma_0 \\ (1, R) & ; \text{if } (\beta, n, \theta) \in \Gamma_1 \\ \left(1, R \min \left\{ \frac{\beta^3(\theta-1)}{\theta((1-\beta^3)(n-1)+1-\theta)}, 1 \right\}\right) & ; \text{otherwise} \end{cases}$$

where  $\Gamma_0 \equiv \{(\beta, n, \theta) | (1-\beta^{3/2})(n-1)+1-\theta > 0\}$  and  $\Gamma_1 \equiv \{(\beta, n, \theta) | (1-\beta^3)(n-1)+1-\theta \leq 0\}$ .

---

<sup>11</sup>That is, in a symmetric allocation,  $Ra_{t-1} + e_t - (n-1)d_t \geq (Ra_{t-1} + e_t)/n$ , given that from the feasibility constraint,  $d_t \leq (Ra_{t-1} + e_t)/n$ .

Let  $(\bar{a}_1, \bar{a}_\phi)$  be as follows:

$$(\bar{a}_1, \bar{a}_\phi) = \begin{cases} \left( \frac{n-1}{n-\theta} \frac{n\beta^3}{n-\theta} (\hat{V}_1 - W_1), \frac{n\beta^3 R}{(n-\theta)\theta} (\hat{V}_1 - W_1) \right) & ; \text{if } \theta < n \\ (\infty, \infty) & ; \text{otherwise} \end{cases}$$

where  $\hat{V}_1 = \frac{1}{(1-\beta^3)^n} (1 - \hat{a}_1 + \theta(\hat{a}_1 - \hat{a}_\phi/R) + \theta^2 \hat{a}_\phi/R)$  and  $W_1 = \frac{1}{(1-\beta^3)^n}$ .

Note that Assumption 1 implies that  $\theta > 1$ . The values  $\hat{a}_1$  and  $\hat{a}_\phi$  represent the equilibrium savings in periods  $T_1$  and  $T_\phi$ , after the allocation becomes stationary (that is, after period  $\hat{t}(t_0)$ ). The values  $\hat{V}_1$  and  $W_1$  represent the present discounted utility of a group in a period  $T_1$  (after the allocation has become stationary) in the best symmetric equilibrium and in the worst, respectively. The values  $\bar{a}_1$  and  $\bar{a}_\phi$  represent the maximum level of savings in periods  $T_1$  and  $T_\phi$  that can be supported in any equilibrium. With these definitions, we can then construct the asset positions,  $\{a_t^*\}$ , that will arise in an efficient and symmetric equilibrium:

**Proposition 1** (Efficiency with No Borrowing). *Suppose that Assumption 1 holds. Then, an efficient and symmetric allocation after default is generated from the following asset positions  $\{a_t^*\}_{t=t_0}^\infty$ :*

$$a_t^* = \begin{cases} 0 & ; t \in T_{\phi^2} \\ \hat{a}_1 & ; t \geq \hat{t}(t_0), t \in T_1 \\ \hat{a}_\phi & ; t \geq \hat{t}(t_0), t \in T_\phi \\ \min\{Ra_{t-1}^* + 1, \bar{a}_1\} & ; t < \hat{t}(t_0), t \in T_1, \\ \min\{Ra_{t-1}^*, \bar{a}_\phi\} & ; t < \hat{t}(t_0), t \in T_\phi; \end{cases}$$

with initial condition  $a_{t_0-1}^* = a_{t_0-1}$ .

### 3.1.1 Discussion of Results With No Borrowing

This subsection explains how the results from Proposition 1 were obtained.

The sustainability constraints (SC) for all periods  $t \geq t_0$  can be written as:<sup>12</sup>

$$\frac{\theta(\theta-1)}{n} a_{t+1}/R + \beta^3 (\hat{V}_1 - W_1) \geq \frac{n-\theta}{n} a_t, \text{ for } t \in T_1 \quad (2)$$

$$\beta^3 (\hat{V}_1 - W_1) \geq \theta \frac{n-\theta}{n} a_t, \text{ for } t \in T_\phi/R \quad (3)$$

<sup>12</sup>The constraints in periods  $T_{\phi^2}$  are automatically satisfied given that savings are zero.

where I have used that the allocation becomes stationary after  $\hat{t}(t_0)$  and hence, the future continuation values of the best equilibrium and the worst are given by  $\hat{V}_1$  and  $W_1$  from Definition 4.

Note that constraints (2) and (3) impose upper bounds on the assets (i.e. saving constraints), and a higher surplus,  $\hat{V}_1 - W_1$ , implies higher bounds. This last point is intuitive, if there is a larger difference between the best equilibrium value,  $\hat{V}_1$  and the worst  $W_1$ , then more cooperation can be sustained (i.e., higher savings). Constraints (2) and (3) can then be used to deliver the upper bounds  $\bar{a}_1$  and  $\bar{a}_\phi$ .

Once the allocation has become stationary, the constraints (2) and (3) can be written as:

$$\theta(\theta - 1)a_\phi/R \geq ((1 - \beta^3)(n - 1) + 1 - \theta)a_1 \quad (SC_0)$$

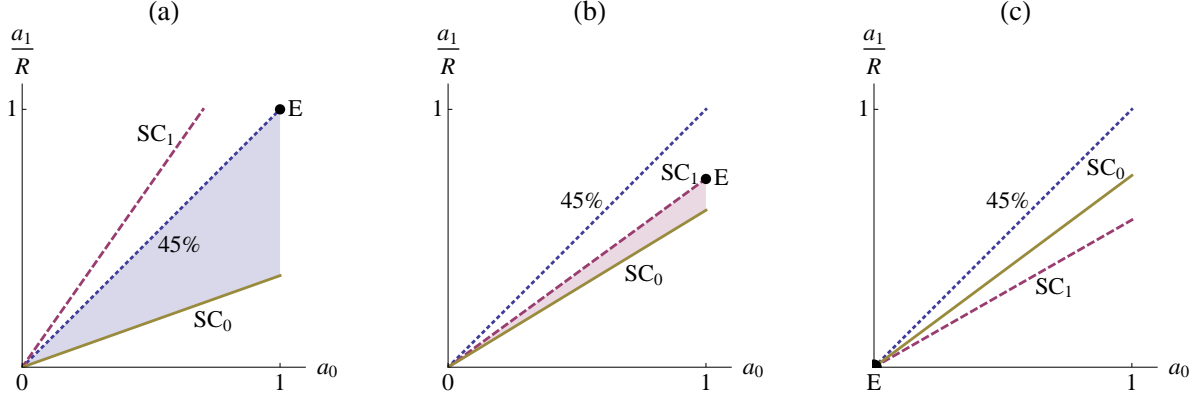
$$\beta^3(\theta - 1)a_1 \geq \theta((1 - \beta^3)(n - 1) + 1 - \theta)a_\phi/R \quad (SC_1)$$

And the problem of finding the efficient and symmetric allocation (after it has become stationary) is then equivalent to finding the values for  $a_1$  and  $a_\phi$  that maximize the present discounted utility of a group in a period  $T_1$ , subject to the constraints above. That is, the problem is to solve:

$$(1 - \beta^3)\hat{V}_1 = \max_{0 \leq a_\phi \leq R a_1 \leq 1} \frac{1}{n} \{1 - a_1 + \theta(a_1 - a_\phi/R) + \theta^2 a_\phi/R\} \text{ s.t. } (SC_0) \text{ and } (SC_1). \quad (\text{P})$$

The solution to Problem P is given by the values  $\hat{a}_1$  and  $\hat{a}_\phi$  stated in Definition 4. Figure 1 illustrates the results. The sustainability constraints are linear in the  $(a_1, a_\phi/R)$ -space and are denoted by  $SC_0$  and  $SC_1$ . The first panel (a) illustrates a situation in which the first-best level of savings is sustainable. The second panel (b) illustrates a situation in which some (strictly positive) level of saving is sustainable but not the first-best level. The third panel (c) illustrates a situation in which no savings are sustainable.

Consider, for example, the case in panel (c). Note that it is not just the temptation to consume the resources in periods  $T_1$  (as given by equation  $SC_0$ ) that generates the no-savings result in this case. After all, if the groups could commit to saving the resources in periods  $T_\phi$  to the next period  $T_{\phi 2}$ , then the first best level of savings could be sustained (that is,  $a_0 = 1$  and  $a_1 = R$  satisfy  $(SC_0)$ ). Hence, the inability to commit to save the resources in periods  $T_\phi$  is what generates the break-down: any stationary saving level that satisfies the sustainability constraints for periods  $T_1$ ,  $(SC_0)$ , violates the sustainability constraint for periods  $T_\phi$ ,  $(SC_1)$ . In the next section I show that external borrowing can (imperfectly) substitute for this commitment to save in periods  $T_\phi$ , and this is the reason why having access to future borrowing is valuable enough as to make debt sustainable.



**Figure 1:**  $SC_0$  denotes the sustainability constraints at  $T_1$  and  $SC_1$ , the sustainability constraints at  $T_\phi$ . The feasibility constraint restricts the choices to lie in the triangle formed by the 45 degree line, the horizontal axis and the vertical line  $a_1 = 1$ . The shaded area represents the area of stationary equilibria. The point  $E$  shows the position of the efficient symmetric sustainable allocation. Panel (a) illustrates a situation in which the first best efficient level of savings is achieved. Panel (b) illustrates a situation with intermediate values of savings. Panel (c) illustrates a case in which the worst equilibrium is also the unique equilibrium.

## 4 Sovereign Borrowing

The previous section described the equilibrium after default, when the country had no access to external borrowing. In this section, the possibility of borrowing from foreign creditors is introduced. In what follows, I restrict attention to simple debt allocations:

**Definition 5.** A **stationary debt contract** is a triplet  $(b, a_1, a_\phi)$  such that  $1 \geq 1 - Rb \geq a_1 \geq a_\phi/R \geq 0$ . The **symmetric allocation of a stationary debt contract**  $(b, a_1, a_\phi)$  is a sequence of fiscal provisions to all groups,  $\{d_t^1, d_t^2, \dots, d_t^n\}_{t \geq 0}$ , such that (i) every group receives the same level of provisions at all times,  $d_t^i = d_t^j$  for  $i, j \in I$ , and (ii) the values of  $-b$ ,  $a_1$  and  $a_\phi$  denote how much the government saves in periods  $T_{\phi^2}$ ,  $T_1$  and  $T_\phi$  respectively. That is  $d_t^i = \frac{1}{n}(Ra^{t-1} - a^t + e_t)$  for all  $t \geq 0$  and  $i \in I$ , and

$$a^t = \begin{cases} -b & ; \text{if } t \in T_{\phi^2}, \\ a_1 & ; \text{if } t \in T_1, \\ a_\phi & ; \text{if } t \in T_\phi, \end{cases}$$

with  $a_{-1} = 0$ .

The contracts above are special in the following critical manner: the government borrows only in the periods when marginal utility is highest. In particular, the government cannot

borrow in any other period. As I will argue below, this aspect of the debt contract is the essential component that guarantees its repayment.

Note that the utility of any group at the beginning of a period in  $T_1$  (the period where debt must be repaid) is given by:

$$V_1^* = \frac{(1 - Rb - a_1) + \theta(a_1 - a_\phi/R) + \theta^2(a_\phi/R + b/R^2)}{n(1 - \beta^3)} \quad (4)$$

Similarly, one can compute the utility of any group at the beginning of a period  $T_\phi$ :

$$V_\phi^* = \frac{\beta^3(1 - Rb - a_1) + \theta(a_1 - a_\phi/R) + \theta^2(a_\phi/R + b/R^2)}{n(1 - \beta^3)\beta} \quad (5)$$

One can immediately see [Bulow and Rogoff \(1989\)](#)'s result arising from equation (4). At any time in  $T_1$ , when the country must pay its debts, consider the symmetric allocation arising from the following alternative strategy: the government defaults, and saves  $a_1 + Rb$  in periods  $T_1$ , saves  $a_\phi + R^2b$  in periods  $T_\phi$ , and saves nothing in periods  $T_{\phi^2}$ . This allocation uses no debt, but generates a payoff to all groups that is strictly higher than  $V_1^*$  (under Assumption 1). That is, a strategy of defaulting and choosing an appropriate level of savings can generate payoffs that dominate the original debt allocation. This means that all groups could increase their utility by defaulting, if they could commit to saving accordingly. This immediately implies that default will necessarily occur in a single domestic agent world and no level of debt would be repaid. With multiple domestic groups, however, it is necessary to check, at a minimum, that this alternative allocation involve levels of savings that are consistent with an equilibrium of the game after default. As I will show below, this is not always the case.

The next definition states the conditions for an allocation to be sustainable:

**Definition 6.** *A stationary and symmetric debt allocation,  $d$ , is **sustainable** if*

(i) **(Unilateral deviations)** *it is the case that*

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_\tau d_\tau \geq \phi_t \max \left\{ Ra_{t-1}^+ + e_t - (n-1)d_t, \frac{Ra_{t-1}^+ + e_t}{n} \right\} + \sum_{\tau=t+1} \beta^{\tau-t} \phi_\tau \frac{e_\tau}{n}; \forall t, \quad (6)$$

(ii) **(Multilateral deviations)** *for all  $t$  and for any subgame-perfect allocation after de-*

*fault,  $\hat{d}$ , of the game starting at  $t$  with assets  $a_{t-1}^+$ , there exists an  $i \in I$  such that*

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau} > \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} \hat{d}_{\tau}^i. \quad (7)$$

where  $a_{t-1}^+ = \max\{a_{t-1}(d), 0\}$ .

Part (i) in the definition guarantees that the debt allocation is subgame perfect. That is, a group cannot gain by unilaterally deviating from the equilibrium allocation, if this behavior triggers the worst equilibrium allocation in the following period. The value of  $a_{t-1}^+$  represents the asset position that the government has after choosing to default in period  $t$ . Part (ii) checks that there are no gains in multilateral deviations: a debt allocation is sustainable if at all times there is no sustainable allocation of the continuation game after default that Pareto dominates the continuation of the debt allocation. This last condition is in the spirit of Bulow and Rogoff's original argument: if all groups were to find jointly optimal to default and coordinate on a better equilibrium with no borrowing, then debt should not be considered sustainable.

Let me briefly discuss part (ii), as it is an assumption in addition to the usual subgame perfection requirement stated in part (i). If we were to impose just part (i), then sovereign debt could be valuable to the groups because it can be used as a trigger mechanism that allows them to sustain cooperation beyond what is achieved in the worst equilibrium. To see this consider a situation where something better than the worst equilibrium is sustainable in the game after default. Then, it is possible to construct a debt allocation so that part (i) is satisfied with a debt contract that prescribes a sufficiently small amount of borrowing. The role of sovereign debt in this case is just to deliver the trigger that can sustain the cooperation in savings that improves upon the worst equilibrium. This example raises two questions. First, why would the groups not coordinate in a better equilibrium after default? If they could, then this will undermine the argument for debt repayment. And second, is the value of sovereign debt just related to this ability to serve as a trigger, or is it also related to the fact that debt is a mechanism that allows the transfer of resources intertemporally? Part (ii) in the definition guarantees that debt will be repaid even if domestic groups are capable of coordinating their equilibrium play after default. As a result, sovereign debt is valuable not because it can serve as a trigger (as those are already allowed for in the considered deviations), but because it allows the groups to transfer resources across time in a way they cannot otherwise replicate.

In the Appendix, I show that by focusing on symmetric debt allocations, constraint (7) can be relaxed: when trying to determine whether a symmetric allocation with debt

is sustainable, it is sufficient to ask whether it is subgame perfect, and whether it is not Pareto dominated by the symmetric and efficient sustainable allocation of the game with no borrowing. The next Lemma summarizes the sustainability conditions for a stationary debt allocation:

**Lemma 3** (Sustainability). *Suppose that Assumptions 1 holds. A stationary debt allocation characterized by  $(a_1, a_\phi, b)$  is sustainable if and only if*

$$V_1^* \geq \left(1 - \frac{(n-1)(1-Rb-a_1)}{n}\right) + \beta^3 W_1 \quad (\tilde{SC}_0)$$

$$\beta V_\phi^* \geq \theta \left(a_1 - \frac{(n-1)(a_1 - a_\phi/R)}{n}\right) + \beta^3 W_1 \quad (\tilde{SC}_1)$$

$$V_1^* \geq \hat{V}_1 \quad (DC_0)$$

$$\beta V_\phi^* \geq \frac{\theta}{n}(a_1 - \min\{a_1, \bar{a}_\phi/R\}) + \frac{\theta^2}{n} \min\{a_1, \bar{a}_\phi/R\} + \beta^3 \hat{V}_1 \quad (DC_1)$$

where  $V_1^*$  and  $V_\phi^*$  are given by equations (4) and (5); and  $\bar{a}_\phi$ ,  $\hat{V}_1$  and  $W_1$  are given by Definition 4.

In the above Lemma, constraints  $(\tilde{SC}_0)$  and  $(\tilde{SC}_1)$  impose the requirement that the allocation be subgame perfect, that is condition (6) of Definition 6. These constraints are the analogs of constraints  $(SC_0)$  and  $(SC_1)$  in the game after default. Constraints  $(DC_0)$  and  $(DC_1)$  impose the requirement that along the equilibrium path, the allocation is not dominated by the efficient allocation after default (this is condition (7) of Definition 6). It is not necessary to check the constraints for periods  $T_{\phi^2}$ , because there are no savings in those periods, and the country is a net receiver of funds from abroad.

## 4.1 Sustainability of Borrowing

An amount of debt  $b$  is said to allow for a sustainable debt allocation if there exists a stationary debt allocation  $(a_1, a_\phi, b)$  that is sustainable. The following proposition, the main result of this paper, characterizes the maximum level of debt that is sustainable:

**Proposition 2.** *Suppose Assumption 1 holds. Let  $\bar{b}$  be the maximum amount of debt that allows for a sustainable debt allocation. The following holds:*



(a) If  $(\beta, n, \theta) \in \Gamma_0$  and  $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ , then  $\bar{b}$  is

$$R\bar{b} = \begin{cases} 1 & ; \text{for } 1 < R^3 \leq \underline{R}_1 \\ \frac{\theta^2 - (1 - \beta^3)(n - 1) - 1}{\theta^2} \frac{R^3}{R^3 - 1} & ; \text{for } \underline{R}_1 \leq R^3 \leq \bar{R}_1 \\ 0 & ; \text{otherwise.} \end{cases}$$

where

$$1 < \underline{R}_1 = \frac{\theta^2}{1 + (n - 1)(1 - \beta^3)} < \bar{R}_1 = \frac{\theta^2 - \theta}{1 + (1 - \beta^3)(n - 1) - \theta},$$

(b) If  $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$  and  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ , then  $\bar{b}$  is

$$R\bar{b} = \begin{cases} 1 & ; \text{for } 1 < R^3 \leq \underline{R}_2 \\ \left[ \frac{(\theta n - 1)(1 - \beta^3) + 1 - \theta^2}{(1 + (1 - \beta^3)(n - 1) - \theta)\theta} \right] \frac{\theta - 1}{\theta} \frac{R^3}{R^3 - 1} & ; \text{for } \underline{R}_2 \leq R^3 \leq \bar{R}_2 \\ 0 & ; \text{otherwise.} \end{cases}$$

where

$$1 < \underline{R}_2 = \theta \left( 1 + \frac{\beta^3(\theta - 1)^2/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} \right)^{-1} < \bar{R}_2 = \frac{\theta(n - 1)}{n - \theta},$$

(c) For all other  $(\beta, n, \theta)$ ,  $\bar{b} = 0$ .

Finally, any level of debt  $b \in (0, \bar{b}]$  allows for a sustainable debt allocation.

To understand the result, consider part (a) of the Proposition. The condition  $(\beta, n, \theta) \in \Gamma_0$  guarantees that zero savings is the unique equilibrium after default, a result that follows from Proposition 1. The condition  $\theta^2 > 1 + (1 - \beta^3)(n - 1)$  guarantees that the constraints  $(SC_0)$  and  $(SC_1)$  are as in the panel (c) of Figure 2: that is, as discussed in subsection 3.1.1, the reason why zero savings emerged as the unique equilibrium of the game after default is because of the binding constraint  $(SC_1)$ , which implies that groups have too strong incentives to spend the resources available in periods  $T_\phi$ . Consider then the decision to repay the debt,  $Rb$ , in a period  $T_1$ . The benefit of doing so is related to the ability of the country to borrow  $b$  in the next period  $T_{\phi^2}$ . That is, it is as if debt repayment allows the country to move resources (ex-post) from periods  $T_1$  to the following periods  $T_{\phi^2}$ . In this sense, debt becomes a savings technology, but note that it is an inefficient one: the implied interest rate is negative. Hence, one may expect that saving instead of repayment is more efficient ex-post. However, *the advantage of using debt rather than savings lies in the fact that using debt bypasses periods  $T_\phi$ , which are periods when incentives are too tight*. When the interest rate is sufficiently low, the ex-post efficiency losses of using debt to transfer resources across time vanishes, while

the incentive benefits of bypassing periods  $T_\phi$  may remain significant. It is in this parameter range that debt is valued ex-post by the groups and repayment is guaranteed.<sup>13</sup>

The following corollary uses the proposition above to show that borrowing up to any level  $b < 1$  is sustainable for a non-empty parameter space if the interest rate is sufficiently close to 1:

**Corollary 1.** *Suppose Assumption 1 holds. If  $(\beta, n, \theta)$  are such that:*

$$1 + (1 - \beta^3)(\theta n - 1) > \theta^2 > 1 + (1 - \beta^3)(n - 1) \quad (8)$$

*then for any  $b \in (0, 1)$ , there exists a value  $\bar{R} > 1$  so that for all  $1 < R < \bar{R}$ , there exists a stationary debt allocation  $(a_1, a_\phi, b)$  that is sustainable. Alternatively, if (8) does not hold, then no positive debt level allows for a sustainable stationary debt allocation for any  $R > 1$ .*

Figure 2 plots the results of Corollary 1. The shaded area represents the region of the state space for which repayable allocations exist under any credit line, as long as  $R$  is sufficiently small. The area denoted by  $A$  is the subset where the worst equilibrium of the game with no borrowing is also the best, and the area denoted by  $B$  is the subset where the best equilibrium of the game with no borrowing is not the worst, but first-best efficiency cannot be achieved.

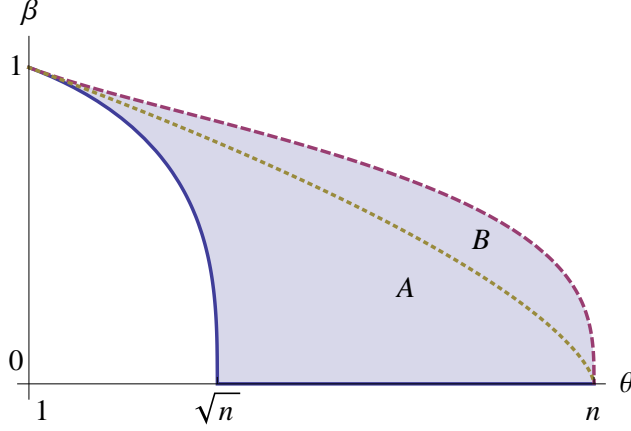
Note that as  $\beta$  goes to one, the set of  $\theta$ 's under which a repayable allocation exists vanishes: as  $\beta$  goes to one, the domestic groups are sufficiently patient that first-best efficiency can always be achieved, and no borrowing can be sustained. In similar fashion, for a given  $\beta < 1$ , when  $\theta$  is very small, no borrowing can be sustained, and no savings are ever done: the benefits of the inter-temporal transfers are not sufficient to generate either. When  $\theta$  is high, the benefits of the inter-temporal transfer are large enough to guarantee that the groups, on their own, can achieve it, and hence the ability to borrow disappears. It is for intermediate values of  $\theta$  that borrowing can be sustained. This is the parameter space where the benefits of cooperation are not sufficiently high to guarantee efficient savings, but not sufficiently low so that borrowing is not valued.

## 5 Discussion

In what follows I briefly discuss a few issues related to the sustainability result.

---

<sup>13</sup>A similar argument applies for part (b). Part (c) shows that when either (i) first best savings can be sustained after default, or (ii) the incentive constraint ( $SC_0$ ) are too tight, then no borrowing can be supported.



**Figure 2:** The shaded area represents the set for which any credit line admits a repayable allocation for some  $R$  close enough to 1. The upper-right boundary (dashed) of the shaded set is  $\theta^2 = 1 + (1 - \beta^3)(\theta n - 1)$ , the lower-left boundary (solid) is  $\theta^2 = 1 + (1 - \beta^3)(n - 1)$ , and the dotted line is  $\theta = 1 + (1 - \beta^{3/2})(n - 1)$ . The set  $A$  are points in  $\Gamma_0$ , so that the worst equilibrium is the unique equilibrium. The set  $B$  are points so that allocations with some strictly positive savings (but below the first best level) are equilibria.

### Unrestricted Access to Credit

One property of the debt contracts consider in the previous sections is that the government can only borrow in periods  $T_{\phi^2}$ . The rest of the periods, the financial markets do not lend. To emphasize that this feature is critical, let us consider the case where the government borrows  $b$  in period  $T_{\phi^2}$  as before, but could in principle borrow as well a period in advance: that is, at  $T_{\phi}$ , the government could borrow up to  $b/R$ . Let us consider the effect on the sustainability constraint ( $\tilde{SC}_1$ ). In this case, the constraint becomes:

$$\beta V_{\phi}^* \geq \theta \left( a_1 - (n - 1) \frac{(a_1 - a_{\phi}/R)}{n} + b/R^2 \right) + \beta^3 W_1$$

and were it is allowed for  $Ra_1 \geq a_{\phi} \geq -b/R$  (that is, that in equilibrium, the government could borrow a fraction in advance).

This is the same constraint as before, except that now, the deviating group in period  $T_{\phi}$  can force the government to borrow at most  $b/R$  from financial markets. Note that the constraint in periods  $T_1$  remains unaffected:

$$V_1^* \geq \left( 1 - (n - 1) \frac{(1 - Rb - a_1)}{n} \right) + \beta^3 W_1$$

Using the values for  $V_1^*$  and  $V_\phi^*$ , the equations above can be written as:

$$\begin{aligned} \beta^3(\theta - 1) \left( a_1 + \frac{b}{R^2} \right) - \theta((1 - \beta^3)(n - 1) + 1 - \theta) \frac{a_\phi + b/R}{R} &\geq (R^3 - 1)\beta^3 \frac{b}{R^2} \\ \theta(\theta - 1) \frac{a_\phi + b/R}{R} - ((1 - \beta^3)(n - 1) + 1 - \theta) \left( a_1 + \frac{b}{R^2} \right) &\geq (R^3 - 1)(n(1 - \beta^3) + \beta^3) \frac{b}{R^2} \end{aligned}$$

Suppose that there exist a solution to the above set of inequalities. Note that the right hand side of both inequalities is non-negative, so it follows that the alternative allocation that involve saving levels of  $(a_1 + b/R^2) \in [0, 1]$  in period  $T_1$  and  $(a_\phi + b/R) \in [0, Ra_1]$  in period  $T_\phi$ , satisfy the sustainability conditions of game after default (as given by equations  $(SC_0)$  and  $(SC_1)$ ). Hence this alternative allocation is an equilibrium after default. However, this allocation dominates the debt allocation as of times  $T_1$  given that  $R > 1$ , and the groups will choose it rather than repaying the country's debt. Summarizing, the type of debt contracts where the country could also borrow in intermediate periods does not admit a sustainable debt allocation. It is, then, a fundamental feature of sustainable debt contracts that credit be restricted in intermediate periods.

### Private savings

If the domestic groups could save on their own, and the return on their assets could not be expropriated by the government or other groups, then the first-best savings-only allocation would be an equilibrium of the game with no borrowing. In that situation, it is an equilibrium for the groups to divide the endowment equally every time, and do on their own all of the inter-temporal transfers.

### Illiquid assets

Suppose that after default, in addition to saving in the linear technology described above, the government also has access to an illiquid technology for saving: the government can invest in periods  $T_1$  and will get a return of  $R^2$  in the following periods  $T_{\phi^2}$ . Importantly, the illiquid nature of this technology precludes the ability to liquidate it in the intermediate periods  $T_\phi$ . Then,

**Lemma 4.** *With an illiquid technology available after default, no stationary debt allocation is sustainable.*

The above lemma highlights that what is valuable about external borrowing is its illiquid nature: the country should only be able to borrow only when it needs it the most. The inability to issue debt in some periods makes external borrowing a commitment device: it

mitigates over-consumption in periods  $T_\phi$ , and hence, makes external borrowing sustainable because it is valuable. The existence of a more efficient commitment technology eliminates the need to use borrowing as a commitment device, and hence repayment becomes unsustainable.

## 6 Conclusion

This paper extends the reputational reason for sovereign debt repayment originally formalized in [Eaton and Gersovitz \(1981\)](#) by introducing political economy considerations. The main departure with previous work is that the sovereign entity is not assumed to maximize the utility of a representative agent, but instead the political economy interactions that lead to default or repayment are explicitly modeled. The basic structure of the model is based on the tragedy-of-the-commons model introduced by [Tornell and Velasco \(1992\)](#) and [Tornell and Lane \(1999\)](#). The model is simple, but rich enough to demonstrate the importance of the implicit assumption of a government as a single-agent in generating the results of [Bulow and Rogoff \(1989\)](#). The main implication of the current paper is that sovereign debt can be repaid in the absence of any other punishments but the threat of exclusion from future borrowing.

# A Collection of Proofs

## Proof of Lemma 1

Note that if all other groups make infinite demands, then, a given group's payoff is independent of its strategy, and thus the total exhaustion profile is an equilibrium. Note also that, any group can guarantee this payoff by always making infinite demands, independent of the strategies of any other group. Hence, the payoff generated by the total-exhaustion strategy profile is the lowest possible.  $\square$

## Proof of Lemma 2

We consider two cases:

Case 1:  $\beta R\phi < n$ .

First, we show that there exists an efficient allocation that is symmetric such that all groups obtain the same payoff after any history.

Standard arguments imply that the set of sustainable payoffs forms a convex, closed, and bounded set (note that the set of sustainable payoffs is bounded above by the first-best payoffs frontier, which is finite given  $\beta R < 1$ , and below by the zero-consumption allocation).

The symmetric structure of the game then implies that there exists an efficient allocation,  $d$ , that delivers the same time-zero payoff to all groups. Let  $V$  be that payoff, so that  $V_\phi^i = V$  for all  $i \in I$ . Now, let a new allocation  $\bar{d}$  be such that  $\bar{d}_t^i = \sum_j d_t^j/n$ . Given that  $\bar{d}$  is a convex combination of  $d$ , the sustainability constraints will be satisfied, as well as the feasibility constraints. So,  $\bar{d}$  is a sustainable allocation that delivers  $V$  to all players at time 0 (it is efficient) and has  $\bar{d}_t^i$  independent of  $i$  for all  $t \geq t_0$ . So let us proceed then, assuming that  $d$  is symmetric.

Suppose that at some time  $t_1 \in T_{\phi^2}$  and  $t_1 \geq t_0$ ,  $a_{t_1}(d) > 0$ . Consider the lowest  $t_2 > t_1$  for which  $d_{t_2} > 0$ , that is  $t_2$  is the first period after  $t_1$  in which strictly positive consumption is allocated to the groups. Such a period must exist, because otherwise the continuation allocation prescribes zero consumption at all times and sustainability constraints cannot possibly be satisfied in periods  $t_1 + 1$  and beyond.

Consider the following feasible perturbation to the allocation  $d$ : decrease  $d_{t_2}$  by  $\varepsilon > 0$  and increase  $d_{t_1}$  by  $\varepsilon/R^{t_2-t_1}$ , while keeping all other  $d_\tau$  the same. Note that this change implies that  $a_\tau$  decreases by  $n\varepsilon/R^{t_2-\tau}$  for  $t_1 \leq \tau < t_2$ , while remaining constant everywhere else. The implied change at time  $t_0$  utility of the perturbation is  $\beta^{t_1-t_0}\varepsilon(\phi^2 - \phi_{t_2}(R\beta)^{t_2-t_1})/R^{t_2-t_1}$ . This change is positive as  $\phi_{t_2} \leq \phi^2$  and  $\beta R < 1$ . The perturbation thus generates an allocation that Pareto dominates  $d$  as of time  $t_0$  (or for any time  $\tau \in [t_0, t_1]$ ).

Now, consider the impact of the change to  $d$  on the sustainability constraints as of time  $\tau$ , as given by (SC):

- For times  $\tau \geq t_2$ , the perturbation to  $d$  does not affect the sustainability constraints because neither  $d_{t+1}$  nor  $a_t$  are affected for  $t \geq t_2$ .
- For times  $\tau$  such that  $t_0 \leq \tau < t_1$ , the change strictly increases the equilibrium utilities of the groups (by the above showing that the perturbed allocation Pareto dominates  $d$

at time  $t_0$ ), and it does not affect the deviation utility because  $a_\tau$  has not changed for  $\tau < t_1$ .

- For time  $\tau = t_1$ , again, the left-hand side of the sustainability constraint has increased, as in the previous case, while the right-hand side has been reduced as  $a_{t_1}$  was reduced; and thus, the sustainability constraint at time  $t_1$  has been relaxed.
- For times  $\tau$  such that  $t_1 < \tau < t_2$ , note that the left-hand side of the sustainability constraints decreases by  $\phi_{t_2}\beta^{t_2-\tau}\varepsilon$  but the right-hand side also decreases, by an amount equal to  $\phi_\tau n \varepsilon R^{\tau-t_2}$ . The sustainability constraint at time  $\tau$  is relaxed after the perturbation if

$$\phi_{t_2}(\beta R)^{t_2-\tau} < \phi_\tau n \quad (9)$$

holds.

There are three possible cases to consider now. First, if  $t_2 \in T_1$  then condition (9) becomes  $(\beta R)^{t_2-\tau} < \phi_\tau n$ , which holds for all  $\tau$  such that  $t_1 < \tau < t_2$ , as  $\beta R < 1$ ,  $\phi_t \geq 1$  and  $n > 1$ . Second, if  $t_2 \in T_\phi$ , condition (9) holds for all  $\tau$  given that  $(\phi\beta R)(\beta R)^{t_2-\tau-1} < n \leq \phi_\tau n$ , which follows from  $\phi_\tau \geq 1$ ,  $\beta R < 1$ ,  $\tau < t_2$  and  $\phi\beta R < n$ . Finally, special care needs to be taken for the case when  $t_2 \in T_{\phi^2}$ . For this, it is useful to note that when sustainability constraints are holding for times  $t \in T_\phi$  for  $t$  such that  $t_1 < t < t_2$ , then the sustainability constraints also are holding at times  $t - 1$ . To see this, note that the sustainability constraint at time  $t \in T_\phi$  is:

$$\sum_{k=1}^{\infty} \phi_{\tau+k} \beta^k d_{\tau+k} - \sum_{k=1}^{\infty} \phi_{\tau+k} \beta^k \frac{e_{\tau+k}}{n} \geq \phi a_t, \quad (10)$$

while at time  $t - 1 \in T_1$ , using that  $d_t = 0$ , the sustainability constraint becomes:

$$\beta \left( \sum_{k=1}^{\infty} \phi_{\tau+k} \beta^k d_{\tau+k} - \sum_{k=1}^{\infty} \phi_{\tau+k} \beta^k \frac{e_{\tau+k}}{n} \right) \geq a_{t-1}. \quad (11)$$

Now,  $a_{t-1} = a_t/R$  given that  $d_t = 0$  from the budget constraint. The term in brackets in equation (11) is the same as the left-hand side of the equation (10). Given that  $\beta R \phi > 1$  by assumption 1, and  $a_t \geq 0$ , it follows that (10) implies (11). Then for  $t_2 \in T_\phi^2$ , it is sufficient to check that the sustainability constraints hold for  $\tau \notin T_1$  after the perturbation. For  $\tau \in T_{\phi^2}$ , condition (9) becomes  $(\beta R)^{t_2-\tau} < n$ , which holds given that  $\beta R < 1 < n$ . For  $\tau \in T_\phi$ , condition (9) becomes  $(\phi\beta R)(\beta R)^{t_2-\tau-1} < n$ , which again follows from  $\beta R < 1 < \phi\beta R < n$ . Hence the perturbation generates a sustainable allocation that Pareto dominates the original.

Taken together, the above implies that in an efficient and symmetric allocation,  $a_t(d) = 0$  for all  $t \in T_{\phi^2}$ .

Finally, for the stationarity result: let  $d$  be an efficient allocation with symmetric payoffs. From the above result, it is now known that  $d_t + d_{t+1}/R + d_{t+2}/R^2 = 1$  for  $t \in T_1$  and  $t > \hat{t}(t_0)$ . Define  $d_i^* = (1 - \beta^3) \sum_{j=\hat{t}(t_0)}^{\infty} \beta^{3j} d_{3j+i}$  for  $i \in \{0, 1, 2\}$ . Let a new allocation  $\hat{d}$  be such that  $\hat{d}_t = d_{t \bmod 3}^*$  for  $t \geq \hat{t}(t_0)$ , and equal to  $d$  otherwise. Note that by construction  $\hat{d}$  achieves the same payoffs as  $d$  as of time 0. The allocation  $\hat{d}$  is feasible, and the linearity of the sustainability constraints guarantees that it is also sustainable.

Case 2:  $\beta R\phi \geq n$ . Note that the symmetric first-best allocation is stationary after period  $\hat{t}(t_0)$ , is symmetric and implies no-over savings. It also follows that the allocation satisfies the sustainability constraints, as can be seen from equations (2) and (3).

And this completes the proof.  $\square$

## Proof of Proposition 1

Let us consider Problem P, which represents the problem after the allocation has become stationary, that is for  $t > \hat{t}(t_0)$ . For this case we have that the following Lemma:

**Lemma 5.** *The values of  $\hat{a}_1$  and  $\hat{a}_\phi$  as given in Definition 4 solve Problem P.*

*Proof.* If  $(1 - \beta^3)(n - 1) + 1 - \theta \leq 0$ , that is, when  $(\beta, n, \theta) \in \Gamma_1$ , then any positive pair  $(a_1, \tilde{a}_\phi)$  satisfy the sustainability constraints, and thus  $a_1 = a_\phi/R = 1$  is sustainable.

If  $(1 - \beta^3)(n - 1) + 1 - \theta > 0$ , then the sustainability constraints can be rewritten as,

$$\frac{\beta^3(\theta - 1)}{\theta((1 - \beta^3)(n - 1) + 1 - \theta)} \geq \frac{a_\phi}{Ra_1} \geq \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)} \quad (12)$$

If  $\frac{\beta^3(\theta - 1)/\theta}{(1 - \beta^3)(n - 1) + 1 - \theta} < \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)}$ , or equivalently, when  $(\beta, n, \theta) \in \Gamma_0$ , then there are no positive values of  $a_\phi$  and  $a_1$  that are sustainable, and thus the worst equilibrium is also the efficient one.

If  $\frac{\beta^3(\theta - 1)/\theta}{(1 - \beta^3)(n - 1) + 1 - \theta} \geq \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)}$ , then it is optimal to pick  $a_1 = 1$  and let  $a_\phi/R$  to be the minimum of between 1 (feasibility) or the left hand side of equation (12).  $\square$

The above characterizes the efficient allocation for  $t > \hat{t}(t_0)$ .

For  $t \leq \hat{t}$  and  $t \in T_\phi$ , it is optimal to make  $a_t$  as high as possible, implying then that  $a_t = \min\{Ra_{t-1}, \bar{a}_\phi\}$ , where  $\bar{a}_\phi$  is the savings limit imposed by inequality ( $SC_1$ ). Finally, for  $t \leq \hat{t}$  and  $t \in T_1$ : define the set  $H \equiv \{(a_1, a_\phi) \in \mathbb{R}^+ \times \mathbb{R}^+ | a_\phi \leq Ra_1; a_\phi \leq \bar{a}_\phi; a_1 \leq \frac{\theta(\theta - 1)}{n - \theta} a_\phi + \frac{n\beta^3}{n - \theta} (\hat{V}_1 - W_0)\}$ . The set  $H$  contains the pairs of  $a_1$  and  $a_\phi$  that are sustainable according to inequalities ( $SC_0$ ) and ( $SC_1$ ). The efficient values of are given by the highest pair of  $(a_1, a_\phi) \in H$  such that  $a_1 \leq Ra_{t_0-1}$ . The upper frontier of  $H$  is given by  $a_\phi \leq Ra_1$  together with  $a_\phi \leq \bar{a}_\phi$ , and  $a_1 \leq \bar{a}_1$ . Finally, for  $t \leq \hat{t}$  and  $t \in T_{\phi^2}$ , we already know that there are no savings done. Taken together, the results of the Proposition follow.  $\square$

## Proof of Lemma 3

The first two constraints ( $\tilde{SC}_0$ ) and ( $\tilde{SC}_1$ ) follow from applying condition (i) in Definition 6 in periods  $T_1$  and  $T_\phi$ . The last two constraints, ( $DC_0$ ) and ( $DC_1$ ), follow from applying condition (ii) in Definition 6, together with the result in Proposition 1, for periods  $T_1$  and  $T_\phi$ . As stated in the main text, periods  $T_{\phi^2}$  do not need to be consider as, in a stationary and symmetric debt allocation, the government is receiving the maximum possible funds from abroad in those periods, and it is not saving.

## Proof of Proposition 2

Part (a) Let  $\tilde{a}_\phi = a_\phi/R$ . Given that  $(\beta, n, \theta) \in \Gamma_0$ , the unique equilibrium of the savings-only game is the worst, so that  $\hat{V}_0 = W_0$ ; and from Proposition 1, it follows that  $\bar{a}_\phi = 0$ , and the



left-hand side of constraint  $(DC_1)$  becomes:

$$\frac{\theta}{n}a_1 + \beta^3W_0$$

which is less than  $\theta \left( a_1 - \frac{(n-1)(a_1 - \tilde{a}_\phi)}{n} \right) + \beta^3W_0$ . This means that constraint  $(DC_1)$  is therefore implied by  $(\tilde{SC}_1)$ . Similarly, constraint  $(DC_0)$  is implied by  $(\tilde{SC}_0)$ . Rewriting  $(\tilde{SC}_0)$  and  $(\tilde{SC}_1)$ , it follows that the problem of finding the maximal amount of debt that allows for a sustainable allocation is:

$$\begin{aligned} \max_{a_1, \tilde{a}_\phi, \bar{b}} \bar{b} \quad & \text{subject to:} & (13) \\ \theta(\theta - 1)\tilde{a}_\phi + \left[ \frac{\theta^2}{R^3} - 1 - (1 - \beta^3)(n - 1) \right] R\bar{b} & \geq [(1 - \beta^3)(n - 1) + 1 - \theta] a_1 & (\tilde{SC}'_0) \\ \theta [(1 - \beta^3)(n - 1) + 1 - \theta] \tilde{a}_\phi & \leq \left( \frac{\theta^2}{R^3} - \beta^3 \right) R\bar{b} + \beta^3(\theta - 1)a_1 & (\tilde{SC}'_1) \\ 1 & \geq 1 - R\bar{b} \geq a_1 \geq \tilde{a}_\phi \geq 0 \end{aligned}$$

We proceed by cases:

– if

$$R^3 \leq \frac{\theta^2}{1 + (1 - \beta^3)(n - 1)} = \underline{R}_1 \quad (14)$$

then setting  $a_1 = \tilde{a}_\phi = 0$  and  $R\bar{b} = 1$  satisfies inequalities  $(\tilde{SC}'_0)$  and  $(\tilde{SC}'_1)$ . Hence, in this case the maximal amount of debt is  $R\bar{b} = 1$ .

– if

$$R^3 > \frac{\theta^2}{1 + (1 - \beta^3)(n - 1)} = \underline{R}_1 \quad (15)$$

then let us ignore for now constraint  $(\tilde{SC}'_1)$ . It follows that it is efficient to set  $\tilde{a}_\phi = a_1$ , as a higher  $\tilde{a}_\phi$  relaxes  $(\tilde{SC}'_0)$ . From  $(\tilde{SC}'_0)$ , we have that

$$R\bar{b} \leq \frac{\theta^2 - 1 - (n - 1)(1 - \beta^3)}{1 + (n - 1)(1 - \beta^3) - \theta^2/R^3} a_1 \quad (16)$$

together with  $R\bar{b} \leq 1 - a_1$ .

Given that (15) holds and that  $\theta^2 - 1 - (n - 1)(1 - \beta^3) > 0$  by the hypothesis of the Proposition, then it follows that to maximize  $\bar{b}$ , we can set  $a_1 = 1 - R\bar{b}$  and make (16) hold with equality. Using this, the maximal debt is

$$R\bar{b} = \frac{\theta^2 - (1 - \beta^3)(n - 1) - 1}{\theta^2} \frac{R^3}{R^3 - 1}$$

We now need to check that  $(\tilde{SC}'_1)$  holds, and that inequality is equivalent to:

$$R^3 \leq \bar{R}_1 = \frac{\theta^2 - \theta}{1 + (1 - \beta^3)(n - 1) - \theta}$$

However, if  $R^3 > \bar{R}_1$ , (so that the above does not hold) then applying the Fourier-Motzkin elimination method shows that there are no possible solutions as the constraint set of problem (13) is empty for any  $\bar{b} > 0$ .

That  $\bar{R}_1 > \underline{R}_1 > 1$  follows immediately from the definition 1 and the hypothesis  $\theta^2 - 1 - (n - 1)(1 - \beta^3) > 0$ .

Note that if  $\bar{b}$  allows for a sustainable allocation for some  $a_1$  and  $a_\phi$ , then for any constant  $\alpha \in (0, 1)$ ,  $(\alpha\bar{b}, \alpha a_1, \alpha\tilde{a}_\phi)$  satisfies the constraints of problem (13), and thus any level of debt below  $\bar{b}$  allows for a sustainable debt allocation.

Part (b) Let  $\tilde{a}_\phi = a_\phi/R$ . Using that  $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$ , and that  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ , it follows that  $\hat{a}_1 = 1$  and  $\hat{a}_\phi = R\beta^3(\theta - 1)/(\theta((1 - \beta^3)(n - 1) + 1 - \theta)) \leq 1$  by Proposition 1.

Using this, constraint  $(DC_0)$  can be rewritten as:

$$(\theta - 1)(a_1 + \theta\tilde{a}_\phi) \geq - \left( \frac{\theta^2}{R^3} - 1 \right) Rb + \frac{(1 - \beta^3)(n - \theta)(\theta - 1)}{1 + (1 - \beta^3)(n - 1) - \theta} \quad (DC'_0)$$

Note that constraint  $(\tilde{SC}_0)$  is

$$(\theta - 1)(a_1 + \theta\tilde{a}_\phi) \geq - \left( \frac{\theta^2}{R^3} - 1 \right) Rb + (1 - \beta^3)(n - 1)(a_1 + Rb)$$

Using that  $a_1 + Rb \leq 1$  and that  $(n - 1) \leq (n - \theta)(\theta - 1)/(1 + (1 - \beta^3)(n - 1) - \theta)$  (which follows from  $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$ ), it follows that the above inequality is implied by  $(DC'_0)$ , and hence  $(\tilde{SC}_0)$  can be ignored in what follows.

Let me now argue that either  $(\tilde{SC}_1)$  or  $a_1 \geq \tilde{a}_\phi$ , holds with equality. This follows by noting that if  $(\tilde{SC}_1)$  is holding with strict inequality and  $a_1 > \tilde{a}_\phi$ , then one can always increase  $\tilde{a}_\phi$  and such an increase relaxes  $(DC'_0)$  and  $(DC_1)$ .

With this, I can argue that constraint  $(DC_1)$  holds as well. This follows from noting that if either  $(\tilde{SC}'_1)$  holds with equality or  $a_1 = \tilde{a}_\phi$ , it follows that

$$\tilde{a}_\phi = \min \left\{ a_1, \frac{\theta}{(n - \theta)R^3} Rb + \frac{n\beta^3}{\theta(n - \theta)} (V_1^* - W_0) \right\}$$

and  $\tilde{a}_\phi \geq \min\{a_1, \bar{a}_\phi/R\}$ , as  $V_1^* \geq \hat{V}_1$  by  $(DC_0)$ , and  $b \geq 0$ . The value in periods  $T_\phi$  under the debt allocation is then:

$$\beta V_\phi^* = \frac{\theta}{n}(a_1 - \tilde{a}_\phi) + \frac{\theta^2}{n}(\tilde{a}_\phi + b/R^2) + \beta^3 V_1^*$$

and  $\tilde{a}_\phi \geq \min\{a_1, \bar{a}_\phi/R\}$  implies that constraint  $(DC_1)$  is satisfied.

Thus, it suffices to find a pair  $(a_1, \tilde{a}_\phi)$  with  $1 \geq 1 - Rb \geq a_1 \geq \tilde{a}_\phi \geq 0$ , such that inequalities  $(\tilde{SC}'_1)$  and  $(DC_0)$  hold.

Given that either  $(\tilde{SC}'_1)$  or  $a_1 \geq \tilde{a}_\phi$  holds with equality, we can focus on the boundary of set of  $(a_1, \tilde{a}_\phi)$  where both  $(\tilde{SC}'_1)$  and  $a_1 \geq \tilde{a}_\phi$  hold:

$$\tilde{a}_\phi(a_1) = \min \left\{ a_1, \frac{\left(\frac{\theta^2}{R^3} - \beta^3\right) Rb + \beta^3(\theta - 1)a_1}{\theta[(1 - \beta^3)(n - 1) + 1 - \theta]} \right\}$$

Given that  $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$ , the right-hand side of the above increases in  $a_1$ , and hence  $\tilde{a}_\phi(a_1)$  increases in  $a_1$ . Given that the  $(DC_0)$  is relaxed with higher  $a_1$  and  $\tilde{a}_\phi$ , it suffices to focus in cases where  $a_1 = 1 - Rb$ . With this, the constraint  $(DC_0)$  becomes:

$$\begin{aligned} \tilde{a}_\phi = \min \left\{ 1 - Rb, \frac{\beta^3(\theta - 1)/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} + \frac{\left(\frac{\theta}{R^3} - \beta^3\right)}{1 + (1 - \beta^3)(n - 1) - \theta} Rb \right\} \\ \geq \frac{\beta^3(\theta - 1)/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} - \frac{\theta/R^3 - 1}{\theta - 1} Rb \end{aligned} \quad (17)$$

together with  $1 \geq Rb \geq 0$ .

Note that we can rewrite constraint (17) as:

$$\begin{aligned} \min \left\{ 1 - \frac{\beta^3(\theta - 1)/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} - \left(\frac{\theta(R^3 - 1)}{R^3(\theta - 1)}\right) Rb, \right. \\ \left. \left( \frac{\left(\frac{\theta}{R^3} - \beta^3\right)}{1 + (1 - \beta^3)(n - 1) - \theta} + \frac{\theta/R^3 - 1}{\theta - 1} \right) Rb \right\} \geq 0 \end{aligned} \quad (18)$$

with  $1 \geq Rb \geq 0$ .

A necessary condition for there to exist a solution to the above system of inequalities with a strictly positive  $b$  is that:

$$\frac{\theta/R^3 - \beta^3}{1 + (1 - \beta^3)(n - 1) - \theta} + \frac{\theta/R^3 - 1}{\theta - 1} \geq 0$$

which is equivalent to,

$$R^3 \leq \bar{R}_2 = \frac{\theta(n - 1)}{n - \theta} \quad (19)$$

Note also that

$$R^3 \leq \underline{R}_2 = \theta \left[ 1 + \frac{\beta^3(\theta - 1)^2/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} \right]^{-1} \quad (20)$$

implies that  $\bar{b} = 1/R$  satisfies (18) as (20) implies

$$1 - \frac{\beta^3(\theta - 1)/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} - \frac{\theta(R^3 - 1)}{R^3(\theta - 1)} \geq 0.$$

and thus inequality (18) holds with  $Rb = 1$ .

In the case that  $\bar{R}_2 \geq R^3 > \underline{R}_2$ , then the maximum amount of sustainable debt is obtained

when the terms inside the minimization in (18) are equalized. This delivers

$$R\bar{b} = \left[ \frac{(\theta n - 1)(1 - \beta^3) + 1 - \theta^2}{(1 + (1 - \beta^3)(n - 1) - \theta)\theta} \right] \frac{\theta - 1}{\theta} \frac{R^3}{R^3 - 1}$$

Now, note that

$$\begin{aligned} \underline{R}_2 &= \theta \left( 1 + \frac{\beta^3(\theta - 1)^2/\theta}{1 + (1 - \beta^3)(n - 1) - \theta} \right)^{-1} \\ &= \left( \frac{1}{\theta} + \left( 1 - \frac{1}{\theta} \right) \frac{\beta^3(\theta - 1)}{\theta(1 + (1 - \beta^3)(n - 1) - \theta)} \right)^{-1} > 1 \end{aligned} \quad (21)$$

where the last inequality follows from noticing that  $\frac{\beta^3(\theta - 1)}{\theta(1 + (1 - \beta^3)(n - 1) - \theta)} < 1$  by the hypothesis that  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ . Similarly, one can show that  $\bar{R}_2 > \underline{R}_2$ .

Finally note that if there was a solution to (18) with positive  $\bar{b}$ , then any  $b \in (0, \bar{b}]$  also solves equation (18). That is, if  $\bar{b}$  admits a sustainable debt allocation, so does any  $b \in (0, \bar{b}]$ .

Part (c) If  $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$  and  $\theta^2 > 1 + (1 - \beta^3)(\theta n - 1)$ , then first best savings is an equilibrium of the game without debt, and thus, from the arguments stated in the body of the paper, no debt can be sustained for any  $R > 1$ .

If  $(\beta, n, \theta) \in \Gamma_1$ , then first best savings is an equilibrium, and thus, no debt can be sustained for any  $R > 1$ .

Finally, if  $(\beta, n, \theta) \in \Gamma_0$ , and  $\theta^2 < 1 + (1 - \beta^3)(n - 1)$ , then  $(\tilde{S}C'_0)$  cannot be satisfied for any  $R > 1$  and  $b > 0$ .

## Proof of Corollary 1

Proposition 2 part (a) states that, for sufficiently small  $R > 1$ , a sustainable debt allocation exist, as long as  $\theta < 1 + (1 - \beta^{3/2})(n - 1)$  and  $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ . Part (b) states that, for sufficiently small  $R > 1$ , a sustainable debt allocation exists as long as  $1 + (1 - \beta^3)(n - 1) > \theta > 1 + (1 - \beta^{3/2})(n - 1)$  and  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ .

Now, note that  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$  implies that  $\theta < 1 + (1 - \beta^3)(n - 1)$ . And it follows that a sustainable allocation exists, for sufficiently small  $R > 1$ , if (i) when  $\theta > 1 + (1 - \beta^{3/2})(n - 1)$  it also holds that  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ , and (ii) if instead  $\theta < 1 + (1 - \beta^{3/2})(n - 1)$  then it also holds  $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ .

Given that if  $\theta = 1 + (1 - \beta^{3/2})(n - 1)$ , both  $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$  and  $\theta^2 > 1 + (1 - \beta^3)(n - 1)$  hold, the result follows.

## Proof of Lemma 4

To show this, consider the case where the country invests in periods  $T_1$  the endowment in the illiquid asset. In periods  $T_\phi$  and  $T_{\phi^2}$ , there are no sustainability constraints as no savings are done, and the wealth of the country cannot be appropriated as it is placed in an illiquid bond. In periods  $T_1$  the sustainability constraints are:

$$\frac{\theta^2}{1 - \beta^3} \geq 1 + \frac{\beta^3}{1 - \beta^3} \frac{1}{n} \quad (22)$$

which is equivalent to:  $n\theta^2 \geq 1 + (n - 1)(1 - \beta^2)$ . From the proof of corollary 1,  $b$  allows for a sustainable debt allocation if  $\theta^2 \geq 1 + (n - 1)(1 - \beta^3)$ . Given that  $n \geq 2$ , this implies that inequality (22) holds, and that the first-best savings is an equilibrium.

## References

- Acemoglu, Daron, Michael Golosov, and Aleh Tsyvinski**, “Political Economy of Mechanisms,” *Econometrica*, 05 2008, 76 (3), 619–641.
- Aguiar, Mark and Gita Gopinath**, “Defaultable debt, interest rates and the current account,” *Journal of International Economics*, June 2006, 69 (1), 64–83.
- **and Manuel Amador**, “Growth in the shadow of expropriation,” *Quarterly Journal of Economics*, 2011, 126 (2), 651–697.
- , – , **and Gita Gopinath**, “Investment Cycles and Sovereign Debt Overhang,” *Review of Economic Studies*, forthcoming.
- Aizenman, Joshua and Nancy Marion**, “International Reserve Holdings with Sovereign Risk and Costly Tax Collection,” *Economic Journal*, 07 2004, 114 (497), 569–591.
- Alesina, Alberto**, “Credibility and Policy Convergence in a Two-Party System with Rational Voters,” *American Economic Review*, September 1988, 78 (4), 496–805.
- Arellano, Cristina**, “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, June 2008, 98 (3), 690–712.
- Atkeson, Andrew**, “International Lending with Moral Hazard and Risk of Repudiation,” *Econometrica*, July 1991, 59 (4), 1069–89.
- Bulow, Jeremy and Kenneth Rogoff**, “Sovereign Debt: Is to Forgive to Forget?,” *American Economic Review*, March 1989, 79 (1), 43–50.
- Cole, Harold L. and Patrick J. Kehoe**, “The role of institutions in reputation models of sovereign debt,” *Journal of Monetary Economics*, February 1995, 35 (1), 45–64.
- **and** – , “Reviving reputation models of international debt,” *Quarterly Review*, 1997, (Win), 21–30.
- Cole, Harold L, James Dow, and William B English**, “Default, Settlement, and Signalling: Lending Resumption in a Reputational Model of Sovereign Debt,” *International Economic Review*, May 1995, 36 (2), 365–85.
- Dixit, Avinash and John Londregan**, “Political Power and the Credibility of Government Debt,” *Journal of Economic Theory*, September 2000, 94 (1), 80–105.

- , **Gene M. Grossman, and Faruk Gul**, “The Dynamics of Political Compromise,” *Journal of Political Economy*, June 2000, *108* (3), 531–568.
- Easterly, William and Ross Levine**, “Africa’s Growth Tragedy: Policies and Ethnic Divisions,” *The Quarterly Journal of Economics*, November 1997, *112* (4), 1203–50.
- Eaton, Jonathan and Mark Gersovitz**, “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, April 1981, *48* (2), 289–309.
- **and Raquel Fernandez**, “Sovereign debt,” in G. M. Grossman and K. Rogoff, eds., *Handbook of International Economics*, Vol. 3, Elsevier, January 1995, chapter 3, pp. 2031–2077.
- English, William B**, “Understanding the Costs of Sovereign Default: American State Debts in the 1840’s,” *American Economic Review*, March 1996, *86* (1), 259–75.
- Grossman, Herschel I and John B Van Huyck**, “Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation,” *American Economic Review*, December 1988, *78* (5), 1088–97.
- Gul, Faruk and Wolfgang Pesendorfer**, “Self-Control and the Theory of Consumption,” *Econometrica*, 01 2004, *72* (1), 119–158.
- Kletzer, Kenneth M. and Brian D. Wright**, “Sovereign Debt as Intertemporal Barter,” *American Economic Review*, June 2000, *90* (3), 621–639.
- Persson, Torsten and Lars E O Svensson**, “Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences,” *The Quarterly Journal of Economics*, May 1989, *104* (2), 325–45.
- Rose, Andrew K.**, “One reason countries pay their debts: renegotiation and international trade,” *Journal of Development Economics*, June 2005, *77* (1), 189–206.
- Sandleris, Guido M.**, “Sovereign Defaults: Information, Investment and Credit,” 2005 Meeting Papers 21, Society for Economic Dynamics 2005.
- Svensson, Jakob**, “Foreign aid and rent-seeking,” *Journal of International Economics*, August 2000, *51* (2), 437–461.
- Tabellini, Guido**, “The Politics of Intergenerational Redistribution,” *Journal of Political Economy*, April 1991, *99* (2), 335–57.

– **and Alberto Alesina**, “Voting on the Budget Deficit,” *American Economic Review*, March 1990, *80* (1), 37–49.

**Tornell, Aaron and Andes Velasco**, “The Tragedy of the Commons and Economic Growth: Why Does Capital Flow from Poor to Rich Countries?,” *Journal of Political Economy*, December 1992, *100* (6), 1208–31.

– **and Philip R. Lane**, “The Voracity Effect,” *American Economic Review*, March 1999, *89* (1), 22–46.

**Worrall, Tim**, “Debt with potential repudiation,” *European Economic Review*, July 1990, *34* (5), 1099–1109.

**Wright, Mark L. J.**, “Competition and Sovereign Risk,” 2004, (6).